FUPRE Journal of Scientific and Industrial Research
Vol.4, (2), 2020
ISSN: 2579-1184 (Print) ISSN: 2578-1129 (Online)

Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) Control Charts Approaches in Monitoring Crime Rate

[1]Ikpotokin O., [2]Ijie T. and [3]Braimah J. O.
[1,2,3]Department of Mathematics and Statistics
Ambrose Alli University, Ekpoma, Nigeria
[1] ikpotokinosayomore@yahoo.co.uk, [3]ojbraimah2014@gmail.com

Abstract
This paper is aimed at monitoring crime rate using Cumulative Sum (CUSUM) and the Exponentially Weighted Moving Average (EWMA) control charts.The crime data analyzed using these charts were extracted from the record office of the Nigeria Police Zonal Head Quarter, Irrua, Edo state. On monitoring process shift in crime rate using CUSUM and EWMA control charts, control parameters were varied in order to assess their performances. For CUSUM control chart, the decision interval(h)was varied at 2.0, 3.0, 4.0 and 5.0 while the EWMA schemes smoothening constant (w)was varied at 0.1, 0.15, 0.2 and 0.25. The finding of the study shows that CUSUM scheme with decision interval h = 2.0 performed optimally by detecting a shift in the process mean in monitoring of crime rate, while EWMA chart scheme with w = 0.2 performed optimally for EWMA chart schemes. The CUSUM control chartwas also observed to detect a smaller shift in the process mean compared to EWMA chart. Therefore, when a smaller shift in the process mean is desired, the CUSUM control chart scheme should be adopted.

Keywords: **Control Limit, Decision Interval, Crime, Centre Line, Smoothing, Weighted**

1. Introduction

Legally, crimes usually are refers to as acts or omissions forbidden by law that can be punished by imprisonment and/or fine. Murder, robbery, burglary, rape, drunken driving, child neglect, and failure to pay your taxes all are common examples. However, as several eminent criminologists recently have noted the key to understanding crime is to focus on fundamental attributes of all criminal behaviors rather than on specific criminal acts*(Gates, 2015)*. Instead of trying to separately understand crimes such as homicide, robbery, rape, burglary, embezzlement, and heroin use, there is the need to identify what they all have in common.Much past research on crime has been confounded by its focus on these politico-legal rather than behavioral definitions. The behavioral definition of crime focused on, criminality; a certain personality profile that causes the most alarming sorts of crimes. All criminal behaviors involve the use of force, fraud, or stealth to obtain material or symbolic resources. As noted in *(Agnew, 1992)*, criminality is a style of strategic behavior characterized by self-centeredness, indifference to the suffering and needs of others, and low self-control. More impulsive individuals are more likely to find criminality an attractive style of behavior because it can provide immediate gratification through relatively easy or simple strategies. These strategies frequently are risky and thrilling, usually requiring little skill or planning.

Nevertheless,*(Gates, 2015)* hypothesis is that the vast majority of legal crime is committed by individuals. Crime is one of the human security problems confronting humanity across the world. Nationshave grappled to contain the rising incidence of homicide, armed robbery, and kidnap, drugtrafficking, sex trafficking, illegal gun runningand host of others. Nigeria is currently caught in the web of crime dilemma, manifesting in the convulsive upsurgeof both violent and non-violent crimes. Notable in this regard are the rising incidents of armedrobbery, assassination and ransom-driven kidnapping, which are now ravaging the polity like atsunami and spreading a climate of fears and anxieties about public safety. The upsurge of crimehas been ongoing as Nigeria has been on the global crime map since 1980s *(Dambazau, 2007)*. These menace ofcrime for decades are traceable to poverty, poor parental upbringing, and greed amongst theyouth; get rich quick mentality, inadequate crime control model of national security amongothers.Events of past few years show that the spate of crime has assumed a debilitating proportion andrequires the intervention of policy makers in this regard*(Okechukwu, 2011 and Osawe, 2015).*The study is therefore aimed at monitoring crime rate using statisticalcontrol charts;the Cumulative Sum (CUSUM) and the Exponentially Weighted Moving Average (EWMA).

The use of techniques and activities to achieve sustain and improve the quality of a product or service is known as statistical quality control. Also, quality control is concerned with quality of conformance of a product or service *(Muttlak and Al-Sabah, 2003)*. The prime purpose of quality control is to assure that the products or services (processes) are performing in an acceptable manner. Organizations accomplish quality control by monitoring process or product or service outputs using statistical techniques like Cumulative Sum (CUSUM). The practical and pragmatic quality control based operations strategy for a service or manufacturing organization would focus on the principle of quality in design *(Nelson, 2005; Chen and Woodall, 2003)*.Service organizations produce services that are tangible. Usually, the complete services might not be seen or touchedrather it is experienced. Examples include crime control inour communities, state and nation at large. Also, since a service is experienced, perceptions can be highly subjective *(Scrucca, 2011).* In addition to tangible factors, quality of services is often defined by perpetual factors. These include responsiveness to customer needs, courtesy and friendliness of staff, promptness in resolving complaints, and atmosphere. Other areas of quality in services include time; the amount of time a customer has to wait for the service, and consistency; the degree to which the service is the same each time*(Riaz, Abbas and Does, 2011)*.The aim of thisstudy is to monitor crime rate usingthe Cumulative Sum (CUSUM) and the Exponentially Weighted Moving Average (EWMA)control charts.

2. Materials and Methods

The methodologies used in this study are the statisticalcontrol charts known asthe Cumulative Sum (CUSUM) and the Exponentially Weighted Moving Average (EWMA).The data used in this study is a secondary data of crime cases obtain from record office, Nigeria Police Station, Divisional Headquarter, Irrua, Edo State which consist of various recorded crimes such as armed robbery, rape, burglary theft, etc for 2015-2018 periods. In order to facilitate the computational efficiency, *minitab*and *anygeth* statistical softwarewere used in this paper to implement the method.

Statistical control charts method was first developed by Shewhart in the early 30's in his research at Bell Telephone Laboratories and was the first to apply statistical methods to quality control hence the named 'Shewhart Control Chart'*(Jafari and Mirkamali, 2011; Lucas, 2016)*. Quality control charts set standards or goals, which can be called quality for a product,allow a decision whether or not the set goal has been achieved *(Duncan, 1993)*. Two types of variation may occur in quality of a product: assignable and chance causes*(Jiang, Shu and Apley, 2008)*.Moreover*(Duncan, 1993)*mentions that chance variations an inherent part of the process behave in a random manner, and they do not show any defined pattern, rather they follow statistical laws. If only chance causes affect the process, the process is said to be under-control. On the contrary, the variation occurred by assignable causes is much larger than the one by chance causes. Therefore, the existence of assignable causes leads to a process out-of-control. Due to the fact that the variation by assignable causes is not an acceptable one, some actions should be taken in order to eliminate it. Controlcharts are extremely useful tools to detect this undesirable shift in the process, and to determine when to take corrective actions *(Jafari and Mirkamali, 2011)*.

Shewhart control charts ($\overline{X}$ charts) consist of two phases in which the control limits are set and the process is monitored respectively. The limits can be set by using the known population parameters. In case of unknown parameters, samples of size n are selected randomly from the in-control process. Then the mean of the process and the variance of the process mean are estimated by using these samples. The most frequent assumption made while constructing control charts is normality assumption *(Dell and Clutter, 2015)*. However, if it is desired to detect the shift in the population mean, then this assumption will no longer hold. As stated by Shewhart, the skewness and kurtosis of a distribution of sample means approach to those of normal regardless of the underlying population distribution *(Ghobbar and Friend, 2003; Eaves and Kingsman, 2004).*

Shewhart charts have been successful because of their simplicity, in which the ease of decision rule is based only on the examination of the last observed point. In other words, one can say that if it is beyond the control limits of the chart, one must investigate the presence of special causes in the process *(Sparks, 2000)*. But this is also a major disadvantage, as it ignores any information given by the previous sequence of points. This makes the Shewhart graph relatively insensitive to small changes in the process, of the order of 1.5 σ (standard errors) or less *(Montgomery, 2009)*, hence the cumulative sum chart.

2.1The Cumulative Sum(CUSUM) Chart

The CUSUM chart is an improvement of the Shewhart control chart and is most appropriate for recognizing data history, missing feature on simpler graphs, and also for recognizing small changes in processes long before graphing alarms *(Ryu, Wan and Kim, 2010)*.In the CUSUM chart, the decision on the state of the process is based on the accumulated information of several samples, not just the last one *(Zhao, Tsung and Wang, 2005)*. In this way, the small evidence that each sample provides of the state of the process, the faster the signalization of small mismatches *(Wu, Yang, Jiang and Khoo, 2008).* These upper and lower CUSUM statistics are calculated by the equations below:

$$C_i^{+} = \max\left[0, X_i - (\mu_0 + k) + C_{i-1}^{+}\right]$$
$$C_i^{-} = \max\left[0, (\mu_0 + k) - X_i + C_{i=1}^{-}\right]$$

(1)

where the initial values for $C_i^{+} = C_i^{-} = 0$. The reference value k is the half between the value of the mean μ_0 and the value of the mean out of control, which one is keen to detect quickly. That is, the reference value k is determined between the desired value μ_0 and the value of the mean out of statistical control μ_1.

$$C_{srsj}^{+} = \max\left[0, C_{srsj-1}^{+} + (\overline{X}_{srs} - \mu_0) - k\right]$$
$$C_{srsj}^{-} = \max\left[0, C_{srsj-1}^{-} - (\overline{X}_{srs} - \mu_0) - k\right]$$

(2)

Also, C_{srsj}^{+} or C_{srsj}^{-} are the one–sided upper and lower CUSUM, if either C_{srsj}^{+} or C_{srsj}^{-} exceeds the predetermined decision value h **(>0)**, the process is considered to be out of control. Then, C_{srsj}^{+} or C_{srsj}^{-} are equal to zero i.e $C_{srsj}^{+} = C_{srsj}^{-} = 0$. Besides, many users may prefer to use the transformation as

$$Z_{srsj} = \frac{\overline{X}_{srs} - \mu_0}{\sigma_{\bar{x}\,srs}}$$

(3)

This will definitely transform $\overline{X}_{srs}$ to Z_{srsj} that follow standard normal distribution for a subgroup size n, where the process is in control. Where by the standardized CUSUM statistics become.

$$C_{srsj}^{+} = \max\left[0, C_{srsj-1}^{+} + Z_{srsj} - k\right]$$
$$C_{srsj}^{-} = \max\left[0, C_{srsj-1}^{-} - Z_{srsj} - k\right]$$

(4)

The reference value k must be chosen in such a way that the value of the sum $\mu_0 + k\sigma$ or ($\mu_0 - k\sigma$) is situated between the mean of the μ_0 process and the mean displaced (out of statistical control) that is to be evaluated. If the change is expressed in units of standard errors when $\mu_1 = \mu_0$, then k represents half of the magnitude of this change, according to this equation

$$k = \frac{\delta}{2}\sigma = \left|\frac{\mu_1 - \mu_0}{2}\right| \quad (5)$$

where δ is the size of the change to be detected in standard error units (σ), μ_0 is the target value and μ_1, the mean value out of control.

According to *(Montgomery, 2009),* a reasonable value for H is five times the value of the standard error (σ), that is, H = 5σ. Defining H = h and K = kσ, and using h = 4 or h = 5 and k =0.5, will result in a CUSUM with good average run length (ARL) properties against a change in process average of 1.5σ (standard errors). It is possible to identify the variations that occur in ARL with different configurations of h and k. The approximating transitions from the in-control to the out-of-control state with a Markov chain that it is provide a simple but very accurate ARL calculation within 1%− 3% procedure based on an approximating equation *(Zhou, Tsung, and Wang, 2008; Ikpotolin, 2015)*.However, the Cumulative Sum control charts have some drawbacks by assuming equal weights for all data points in the data set, this means that all the data set contributes equally to the signal flagged by the CUSUM chart, this in reality is not the case and the Exponentially Weighted Moving Average (EWMA) control chart was therefore introduced to address this draw back.

2.2 The Exponential Weighted Moving Average (EWMA)

The Exponential Weighted Moving Average (EWMA) chart is used for monitoring process by averaging the data in a way that give less weight to old data as samples are taken and give a high weight to most recent data. It is also is effective in detecting small shifts.The EWMA charting procedure is sometime used to monitor the rate of occurrence of rare events where the time between two successive occurrences is exponentially distributed. This procedure can also be used extensively in time series modeling and forecasting*(Montgomery,2009)*. The EWMA for individual value may be defined as:

$$Z_i = wX_i + (1 - w)Z_{i-1} \quad (6)$$

where $0 < w \leq 1$ and $i = 1,2, \ldots, n$

While Shewhart charts only consider the most recent data point in testing to determine if statistical limits have been exceeded, EWMA charts consider all previous points using a weighing factor that makes the outcome more influenced by recent points *(Abbas, Riaz and Does, 2012)*. The EWMA for the ith subgroup (Z_i) is defined recursively as:

$$Z_t = wX_i + (1 - w)Z_{i-1} \quad (7)$$

where w is the weighted parameter (0 ≤ w ≤1).

If the value of μ is known, $E_O = \mu_O$, otherwise $E_o = X$. Hence, the preceding equation can be rewritten as:

$$Z_t = Z_{i-1} + w(X - Z_{i-1}) \quad (8)$$

which expresses the current EWMA plus the weighted error in the prediction of the current mean based on the previous mean. The EWMA for the ith subgroup can be written as:

$$Z_t = w(1 - w)jX_i - j + (1 - w)^i E_o \quad (9)$$

which expresses the EWMA as weighted average of past subgroup means where the weight decline exponentially as heaviest weight is assigned to the recent subgroup mean.By default, the central line on EWMA chart indicates an estimate for μ, which is computed as:

$$\hat{\mu} = \bar{\bar{X}} = \frac{n_1\bar{X}_1 + \cdots + n_N\bar{X}_N}{n_1 + \cdots + n_N} \quad (10)$$

If $\mu = \mu_o$, the central line indicates the value of μ_o and the limit can be computed as follows.

The control limits are given as

$$UCL = \bar{\bar{X}} + q\sigma\sqrt{\frac{w}{n(2-w)}} \quad (11)$$

$$LCL = \bar{\bar{X}} - q\sigma\sqrt{\frac{w}{n(2-w)}} \quad (13)$$

where Z_t = Exponentially Weighted Moving Average, w= EWMA weighted parameter ($0 < w < 1$), μ_0 = Process mean, X_{ij}= j^{th} measure of i^{th} subgroup, σ = Process standard deviation, n_i = Sample size of i^{th} group, q=sigma limit and $\bar{X}_i$= Mean of measurement in i^{th}subgroup.

The data used for this study consists of monthly crime statistics in Esan Central Local Government Area of Edo state from year 2015 to 2018 as shown in Table 1. The various control chart schemes were implemented by varying their parameters in order to obtain the most efficient scheme to monitor crime rates using crime data collected from record office of Nigeria Police Force, Irrua, Edo state.For the CUSUM charts, the decision interval, h was varied as shown in Tables 2 and out of control shifts detected as shown in Table 3.

3. Results and Discussion

Table 1: Crime Statistics in Irrua, Esan Central L.G.A of Edo State, 2015 – 2018

Month	Year			
	2015	2016	2017	2018
JAN	32	30	39	39
FEB	36	29	34	34
MAR	38	30	41	32
APR	38	30	40	32
MAY	34	32	29	30
JUN	28	29	29	32
JUL	29	28	39	34
AUG	30	28	34	37
SEP	33	33	35	34
OCT	35	35	35	33
NOV	35	35	30	38
DEC	32	33	29	33

Source: Record Office of Nigeria Police, Irrua Divisional Headquarters

Table 2: In-Control Mean (K_2), Out-Of Control Mean (K_1) and Decision (h) for CUSUM

Shift Size	Target Values/Decision Interval		
	K_1	K_2	Decision Interval (h)
f = 1.0	0.5952	0.1350	2
f = 1.2	0.6412	0.0890	3
f = 1.5	0.7103	0.0200	4
f=6	0.8193	0.0087	5

The target value ‘k’ and the reference value ‘h’ were obtained using *anygeth* software.With $k_1 = \mu + f\sigma$ and $k_2 = \mu - f\sigma$ and the sift size, f prefixed by taking into consideration the amount or size of shift (multiple of the standard deviation) to be detected by the chart, the control chart was then developed using Minitab statistical software. More explicitly, f can simply be thought of as the size of variation to be considered assignable by the manager.

Table 3: Summary of Out of Control Shifts Detected

CUSUM Parameter	First Out of Control Signal Detected	Total Out of Control Detected
k=0.3103 h=2	4	10
k=0.2800 h=3	4	9
k=0.1930 h=4	27	3
k=0.0895 h=5	28	1

From Table3, the crime rate in Irrua and its environ is out of control for the four shift sizes (f) and decision interval (h). It can be seen from the table that the smaller the decision interval, the faster an out of control shift is detected and the more the total numbers of shifts are detected.

4.1 Discussion of Results fromCUSUM Control Charts

An attempt to get the best possible CUSUM control chart by comparing various control charts using different decision intervals (h) was made here. The CUSUM control schemes were designed with h = 2.0, 3.0, 4.0 and 5.0. This is to enable comparison and determine which scheme will be more effective in detecting a small shift in the mean of crime recorded in Irrua, Edo State.

The various CUSUM schemes are shown in Figs 1 to 4 below.

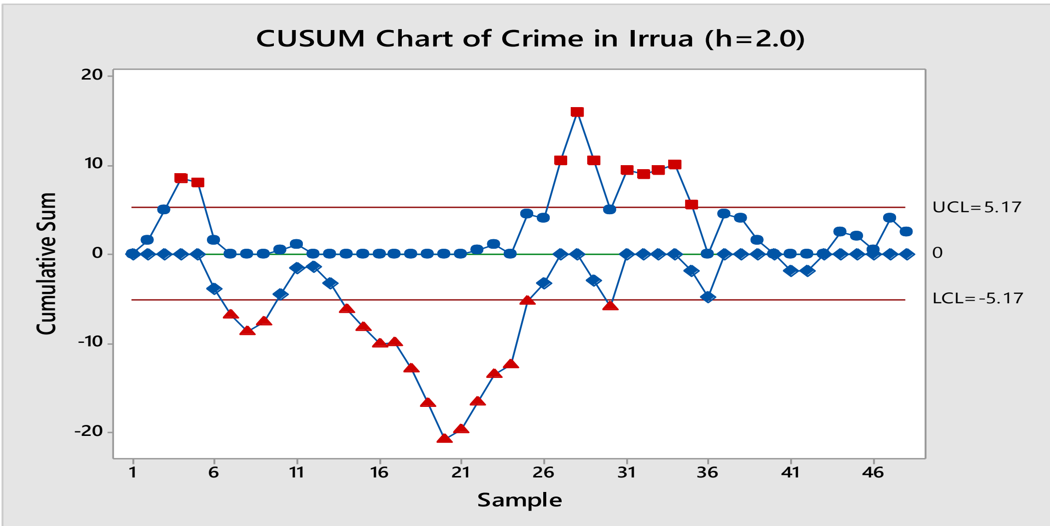

Figure1: CUSUM chart of crimes in Irrua, Edo State with (h = 2.0)

Figure 1 shows the CUSUM chart of crimes recorded in Irrua, Edo State with decision interval (h = 2.0). The chart reveals a total of 10 Out-of-Control cases with the first shift detected at point 4 (circled).

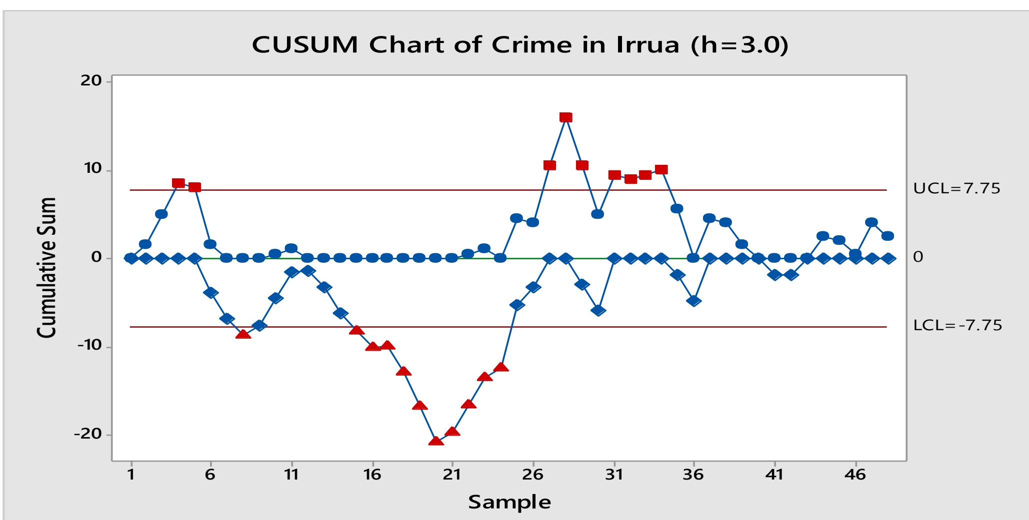

Figure2: CUSUM chart of crimes in Irrua, Edo State with (h = 3.0)

Figure 2 shows the CUSUM chart of crimes recorded in Irrua, Edo State with decision interval (h = 3.0). The chart detected a total of 9 out-of-control cases with the first shift detected at point 4.

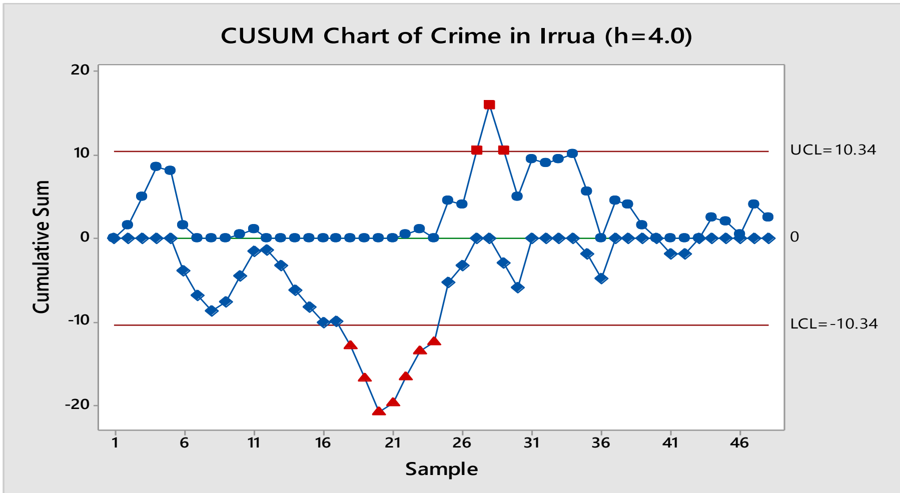

Figure3: CUSUM chart of crimes in Irrua, Edo State with (h = 4.0)

Figure 3 shows the CUSUM chart of crimes recorded in Irrua, Edo State, designed with decision interval (h = 4.0). This chart reveals a total of 3 out-of-control cases with the first shift detected at point 27.

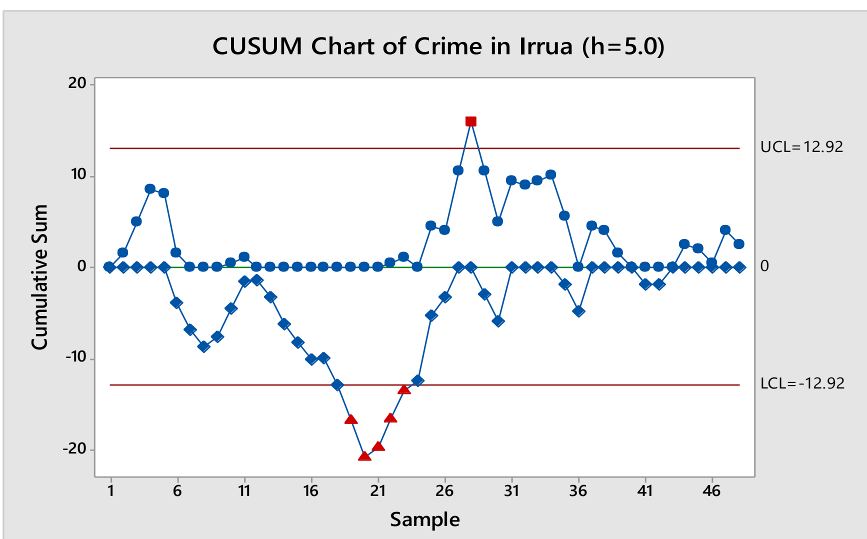

Figure 4: CUSUM chart of crimes in Irrua, Edo State with (h = 5.0)

Figure 4 shows the CUSUM chart of crimes recorded in Irrua, Edo State, designed with decision interval (h = 5.0). This chart reveals a total of 1 out-of-control case which was detected at point 28.

In general, it is observed from the analyses that the Cumulative Sum (CUSUM) control chart with decision interval (h = 2.0) is the best possible scheme for controlling the crime rate in Irrua, Edo State. The scheme detected the earliest shift in process mean with the highest out-of-control case in the data under study.

4.2 Discussion of Results from EWMA Control Charts

The Exponentially Weighted Moving Average (EWMA) control chart was designed using different smoothing constants (w), otherwise known as weights. This was done in order to compare the various schemes and determine the best possible EWMA control chart for monitoring crime rate in Irrua, Edo state. This is usually the chart that detects an early shift in the process mean. The EWMA control schemes were designed with w = 0.1, 0.15, 0.20 and 0.25 as shown in figs 5 to 8.

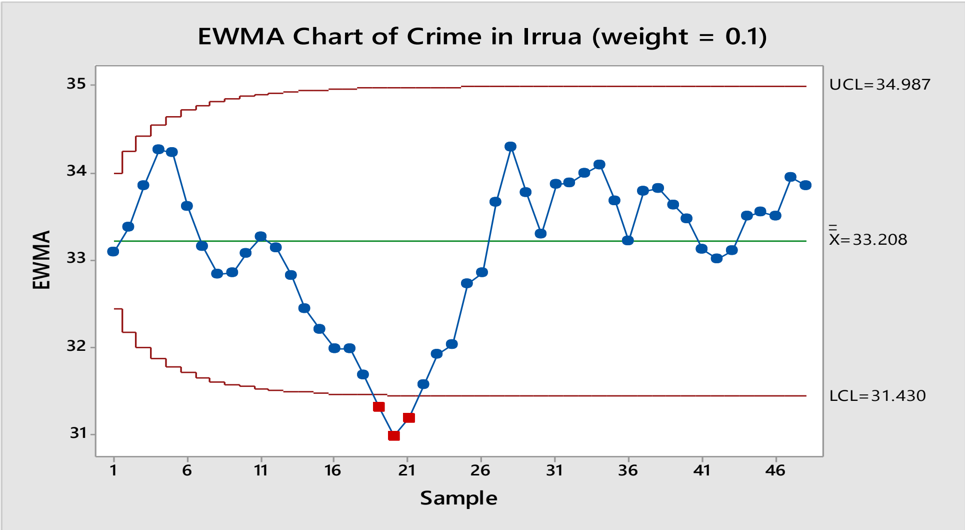

Figure5: EWMA chart of crimes in Irrua, Edo state with (w = 0.1)

Figure 5 shows the EWMA chart of crimes recorded in Irrua, Edo state, designed with decision interval (w = 0.1). This chart shows that the crime control process was in control since there was no plotted point above the upper control limit.

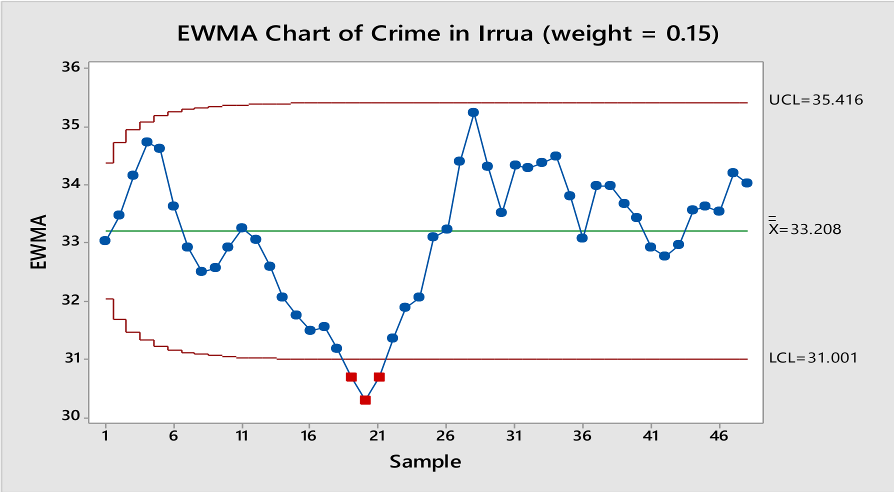

Figure6: EWMA chart of crimes in Irrua, Edo state with (w = 0.15)

Figure 6 shows the EWMA chart of crimes recorded in Irrua, Edo state, designed with decision interval (w = 0.15). This also shows that the crime control process was in control since there was no plotted point above the upper control limit.

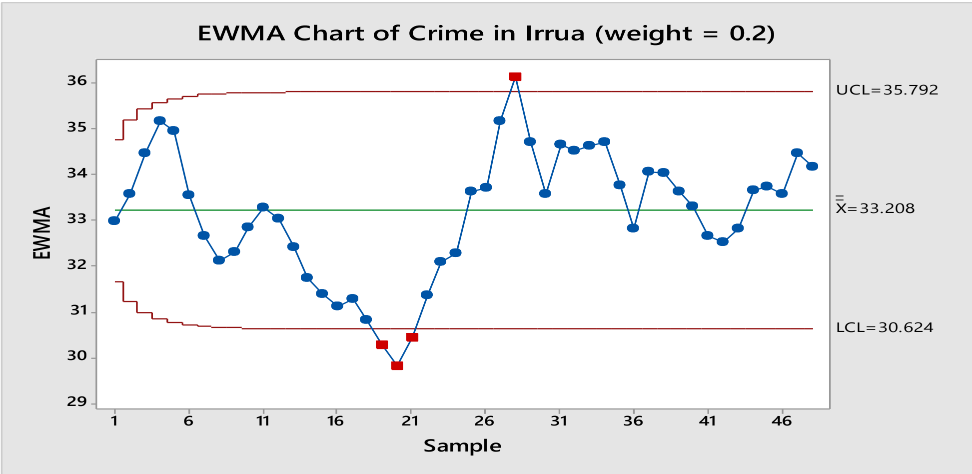

Figure7: EWMA chart of crimes in Irrua, Edo state with (w = 0.2)

Figure 7 shows the EWMA chart of crimes recorded in Irrua, Edo state, designed with decision interval (w = 0.2). This chart detected an out-of-control shift at point 27.

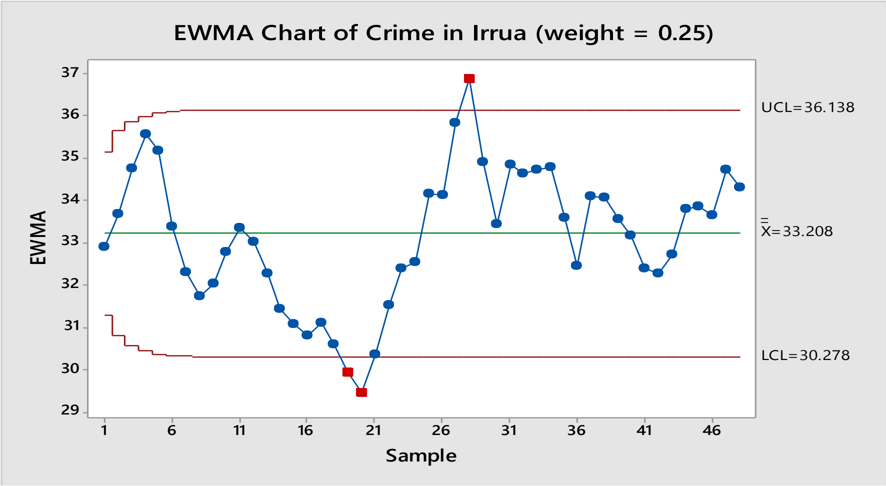

Figure8: EWMA chart of crimes in Irrua, Edo state with (w = 0.25)

Figure 8 shows the EWMA chart of crimes recorded in Irrua, Edo state, designed with decision interval (w = 0.25). This chart detected an out-of-control shift at point 28.

However, it is also observed from the analyses that the various Exponentially Weighted Moving Average (EWMA) control chart with varying smoothening constants (w) do not show much difference in their performance. Generally, on comparing the EWMA and the CUSUM control charts, it is observed that the CUSUM charts perform better that than the EWMA control charts. This is because the CUSUM detects an early and small shift in the process mean than the EWMA control.

Conclusion

From the analysis using the CUSUM and EWMA control charts, it was observed that the crime rates in Irrua, Edo state is out of statistical control. The study further shows that the CUSUM control chart using a decision interval (h) of 2.0 performed better in monitoring the crime rates when a small shift in crime control process is desired than EWMA where there was no significant in detection of out of control signal. What this implies is that EWMA control chart may be misleading that a process is in control where as in the real case, it is out of control. It is noted from the findings of this study that the choice of decision interval(h) for CUSUM chart should be small while the smoothening parameter (w) for EWMA chart should be high when designing these control chart schemes to detect small shift in the process mean.

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