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Pressure and Pressure Derivative of a Horizontal Well Subjected to a Single Edge and Bottom Water Drive Mechanism

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Abstract

A horizontal well test analysis is complicated and difficult to decipher. Most mathematical models for horizontal wells assume that they are completely horizontal and parallel to the reservoir's top and bottom borders. The goal of this study is to develop a mathematical model for a horizontal well completed in an oil reservoir at late time flow period for isotropic reservoirs, where the reservoir is bounded by an edge and bottom constant pressure boundaries for the interpretation of pressure responses, as part of an effort towards correct horizontal well test analysis.Using reservoir and well parameters, Excel software was used to investigate which set of reservoir and well parameters would prolong productivity before steady state set in. The higher the well productivity and the longer it takes to reach steady state, the greater the distance between the well and the x and z-boundaries (constantpressure boundaries).Dimensionless pressure rises as reservoir pay thickness increases, delaying steady-state conditions. When the well length is short, as in this investigation, the dimensionless lateral extent has no direct effect on the dimensionless pressure and derivative. As a result, the distance from the constant pressure boundary, well length, and pay thickness are the most important factors in determining external fluid inversion in the wellbore.

Keywords: Dimensionless Pressure, Derivative, Horizontal Well, Late flow Period, Isotropic Reservoir

1. Introduction

Because of the possibility of many transient flow periods, the pressure behavior of a horizontalwell is significantly more difficult than that of a vertical well. To calculate pressure derivativesand significant characteristics on an oil reservoir, this study examines the pressure derivatives andimportant characteristics of a horizontal well subjected to edge water drive.

The main objective of this study is to determine the dimensionless pressure and derivatives of a late time flow period in a horizontal well subjected to single edge and bottom water drive mechanism using the dimensionless pressure and derivative equations and to investigate the necessary dimensionless parameters in order to identify the parameters that strongly affects dimensionless pressure responses and the minimum time for steady state period to set in.

For years, oil and gas reservoir qualities have been assessed by interpreting pressure data collected during a well test. The stable reservoir pressure measured in shut-in wells is used to calculate reserves in place using material balance calculations. Transient pressure analysis is used to describe reservoir flowing behavior. There have been a number of approaches to interpreting transient testing, but Horner's method is the most well-known and widely utilized by petroleum engineers. Type curves, which depict the pressure response of flowing wells under a variety of reservoir descriptions, were established only a few years ago as the only reliable method of determining the portion of the pressure data that can be analyzed using classic straight line analysis methods.

Because of the availability of reliable pressure data (from electronic pressure gauges) and the development of new computer-aided analysis tools, the quality of well test interpretations has increased substantially in recent years. Various analysts believe type curves are overly simple, overly complex, difficult to discern, and/or time-consuming to use. Identifying straight lines on a pressure versus time graph, on the other hand, is a "ruler technique," which is helpful for hand analysis but overlooks today's incredible computing resources. Furthermore, standard straight line analysis methods do not use all available data, resulting in significant mistakes.

This study presents a method of interpretation based on the examination of the pressure derivative with regard to the time function in use: natural logarithm or Horner/superposition times. Using the type curve matching methodology, this method analyzes the reaction as a whole, from extremely early time data to the latest recorded point. The infinite acting radial flow regime is highlighted by the logarithmic derivative, which is critical for well test interpretation. Because the derivative of pressure vs time is explicitly represented in one term of the diffusivity equation, which is the governing equation for all existing models of transient pressure behavior in well test analysis, it is theoretically satisfying. As a result, the derivative response is more sensitive to integrated small phenomena of interest, and the pressure versus time solutions currently utilized in well test interpretation have a lower derivative response.

2. Methodology

The method employed in this work is to develop an equation for dimensionless pressure drop and dimensionless pressure derivative for horizontal wells located in a reservoir bounded by single edge and bottom water drives. The Gringarten et al (1973) and Adewole et al (2019) for steady state peroid will be utilized for appropriate instantaneous source function.

2.1 *Reservoir Model Description*

The figure1 shows the isotropic rectangular shaped reservoir model under study. It consists of Horizontal well where all the permeability is the same in all directions ($k_x=k_y=k_z$). The horizontal well is designed such thatd$_x<<x_e-D_x$, $d_y=y_e-D_y$ and $d_z>>z_e-D_z$ (Mutili Peter Mutisya et al, 2020).

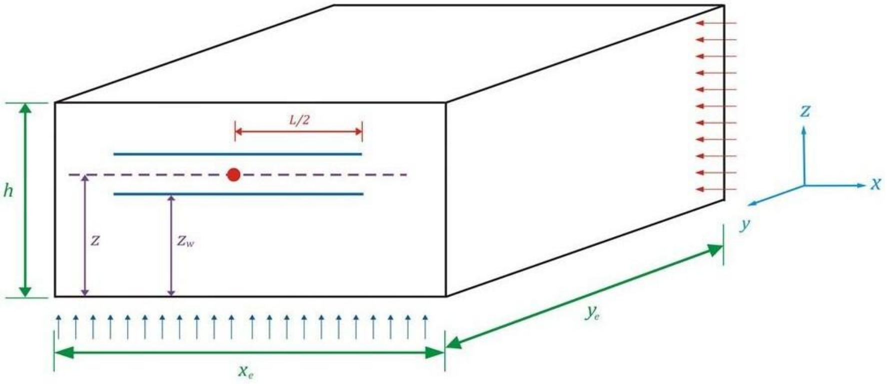

Figure 1: Horizontal well in a reservoir bounded by single edge and bottom water drive.

Where
L= Reservoir Thickness
X=Distance to the boundary in x-direction
Y=Distance to the boundary in Y-direction
Z=Distance to the boundary in Z-direction
X_e = Half the distance to the boundary in X-direction
Y_e=Half the distance to the boundary in Y- direction
Z_e=Half the distance to the boundary in the Z-direction
Z_w=The dimensionless distance from the bottom of the reservoir to the bottom of the wellbore

2.2 Source Functions

Instantaneous source or Green's functions are selected based on the type of boundary condition according to Gringarten et al (1973).

2.3 Mathematical Model Description

Using dimensionless parameters and considering a point source as the intersection of three perpendicular infinite planes, the instantaneous point source function is obtained by Newman's product method, Gringarten et al (1973) as:

$$P_D = 2\pi h_D \int_0^{t_D} s(x_D, t_D) \cdot s(y_D, t_D) \cdot s(z_D, t_D) dt \tag{2.10}$$

Where:

$$s(x_D, t_D) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{1}{n+1} \exp\left(-\frac{(2n+1)^2 \pi^2 t_D}{4x_{eD}^2}\right) \sin\left(\frac{(2n+1)\pi x_{wD}}{2x_{eD}}\right) \cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right) \cos\left(\frac{(2n+1)\pi x_D}{2x_{eD}}\right) \tag{3.11}$$

$$s(y_D, t_D) = \frac{1}{y_{eD}} \left(1 + 2\sum_{n=1}^{\infty} \exp\left(-\frac{n^2 \pi^2 t_D}{y_{eD}^2}\right) \cos\left(\frac{n\pi y_{wD}}{y_{eD}}\right) \cos\left(\frac{n\pi y_D}{y_{eD}}\right)\right) \tag{2.12}$$

And

$$s(z_D, t_D) = \frac{1}{h_D} \sum_{n=1}^{\infty} \exp\left(-\frac{(2n-1)^2 \pi^2 t_D}{4h_D^2}\right) \sin\left(\frac{(2n-1)\pi z_{wD}}{2h_D}\right) \sin\left(\frac{(2n-1)\pi z_D}{2h_D}\right) \tag{2.13}$$

The dimensionless pressure derivative at steady state is given as:

$$P_D' = t_D \frac{\partial P_D}{\partial t_D} \tag{2.14}$$

By Substitution we have;

$$P_D^{'} = \frac{16 t_D}{y_{eD}} \left[\begin{array}{l} \sum_{n=1}^{\infty} \frac{1}{n+1} \exp\left(-\frac{(2n+1)^2 \pi^2 t_D}{4 x_{eD}^2} \right) \sin\left(\frac{(2n+1)\pi x_{wD}}{2 x_{eD}} \right) \cos\left(\frac{(2n+1)\pi x_D}{2 x_{eD}} \right) \\ \cos\left(\frac{(2n+1)\pi x_{eD}}{2 x_{eD}} \right) * \left(1 + 2\sum_{n=1}^{\infty} \exp\left(-\frac{n^2 \pi^2 t_D}{y_{eD}^2} \right) \cos\left(\frac{n\pi y_{wD}}{y_{eD}} \right) \cos\left(\frac{n\pi y_D}{y_{eD}} \right) \right) \\ * \sum_{n=1}^{\infty} \exp\left(-\frac{(2n-1)^2 \pi^2 t_D}{4 h_D^2} \right) \sin\left(\frac{(2n-1)\pi z_{wD}}{2 h_D} \right) \sin\left(\frac{(2n-1)\pi z_D}{2 h_D} \right) \end{array} \right] \qquad (2.15$$

2.4 Dimensionless Expressions for Isotropic Reservoir

$$h_D = \frac{2h}{L}, \; L_D = \frac{L}{2h} \; (2.16)$$

$$X_{wD} = \frac{2X_w}{L} \qquad (2.17)$$

$$Y_{wD} = \frac{2Y_w}{L} \qquad (2.18)$$

$$Z_{wD} = \frac{2Z_w}{L} \qquad (2.19)$$

$$X_{eD} = \frac{2X_e}{L} \qquad (2.20)$$

$$Y_{eD} = \frac{2Y_e}{L} \; (2.21)$$

$$Z_{eD} = \frac{2Z_e}{L} \quad (2.22)$$

$$X_D = \frac{2X}{L} \qquad (2.23)$$

$$Y_D = \frac{2Y}{L} (2.24)$$

$$Z_D = \frac{2Z}{L} \quad (2.25)$$

2.5 Model Parameters

Consider the following numerical data of rock and reservoir characteristics properties in a bounded isotropic reservoir;

$L = 500ft$

$x = x_w = 100ft, y = y_w = 200ft, z = 150ft, z_w = 160ft$

$x_e = 1000ft, y_e = 500ft, z_e = 300ft$

$h = 200ft, d_x = 134ft, d_y = 200ft, d_z = 160ft$

$D_x = 650ft, D_y = 210ft, D_z = 170ft$

K=100mD

3. Results and Discussion

3.1 Results Presentation

3.1.1 The Effect of Well Lengths on PD and PD' for a Given Reservoir Length.

The effect of well length on dimensionless pressure and derivative was investigated, keeping the external lateral extent (XeD) of the reservoir constant at 4 and different values of well lengths (L=500ft, 800ft, 1000ft and 1500ft) were investigated for their effect on dimensionless pressures and derivative. From the Table1, result shows that well length is inversely related to the dimensionless pressure. The higher the value of well length, the smaller the dimensionless pressure. Also, it is pertinent to note that the magnitude of dimensionless pressures derivative increases at early time with increasing well length and decreases as it approaches late time.

Table 1. Dimensionless Pressures and derivative results for varying well length (L) when XeD=4.0

tD	PD, L=500ft	PD', L=500ft	PD L=800ft	PD' L=800ft	PD,L=1000ft	PD' L=1000ft	PD,L=1500ft	PD',L=1500ft
0.0001	4.4464E-05	4.4464E-09	4.441E-05	7.1052E-09	1.41921E-05	8.87007E-09	9.425E-06	1.32539E-08
0.001	4.4156E-05	4.4156E-08	4.363E-05	6.9801E-08	1.38035E-05	8.62717E-08	8.85421E-06	1.24512E-07
0.01	4.1195E-05	4.1195E-07	3.652E-05	5.8437E-07	1.04569E-05	6.53559E-07	4.74063E-06	6.66651E-07
0.1	2.0577E-05	2.0577E-06	6.178E-06	9.8844E-07	6.50978E-07	4.06861E-07	9.17697E-09	1.29051E-08
1	1.9897E-08	1.9897E-08	1.184E-13	1.8949E-13	5.69088E-19	3.5568E-18	6.78156E-36	9.53656E-35
10	1.4217E-38	1.4217E-37	7.939E-91	1.2703E-89	1.4836E-139	9.2724E-138	3.2933E-307	4.6312E-305
100	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0
10000	0	0	0	0	0	0	0	0

3.1.2 The Effect of Dimensionless Reservoir Height on PD and PD’ for a Given Reservoir Length.

The effect of dimensionless pay thickness on dimensionless pressure and derivative was investigated, keeping the external lateral extent (XeD) of the reservoir constant at 4 and 8 with different values of dimensionless pay thickness (hD=0.8, 1 and 2) were investigated for their effect on dimensionless pressures and derivative. The results are presented in Tables 2 and 3 and Figures 2 and 3 respectively.

Table 2: Dimensionless Pressures and derivative results for varying dimensionless pay thickness (hD) when XeD=4

tD	PD, hD=0.8	PD', hD=0.8	PD, hD=1	PD', hD=1	PD, hD=2	PD', hD=2
0.0001	4.446E-05	4.4464E-09	2.85E-05	2.85E-09	7.12E-06	7.12E-10
0.001	4.416E-05	4.4156E-08	2.83E-05	2.83E-08	7.09E-06	7.09E-09
0.01	4.119E-05	4.1195E-07	2.67E-05	2.67E-07	6.81E-06	6.81E-08
0.1	2.058E-05	2.0577E-06	1.51E-05	1.51E-06	4.55E-06	4.55E-07
1	1.99E-08	1.9897E-08	5.1E-08	5.1E-08	8.12E-08	8.12E-08
10	1.422E-38	1.4217E-37	9.73E-33	9.73E-32	2.66E-25	2.66E-24
100	0	0	6.2E-280	6.2E-278	3.8E-200	3.8E-198
1000	0	0	0	0	0	0
10000	0	0	0	0	0	0

Table 3: Dimensionless Pressures and derivative results for varying dimensionless pay thickness (hD) when XeD=8

tD	PD, hD=0.8	PD', hD=0.8	PD, hD=1	PD', hD=1	PD, hD=2	PD', hD=2
0.0001	2.223E-05	2.2234E-09	1.42E-05	1.42E-09	3.56E-06	3.56E-10
0.001	2.21E-05	2.2101E-08	1.42E-05	1.42E-08	3.55E-06	3.55E-09
0.01	2.081E-05	2.0813E-07	1.35E-05	1.35E-07	3.44E-06	3.44E-08
0.1	1.142E-05	1.1418E-06	8.4E-06	8.4E-07	2.53E-06	2.53E-07
1	2.818E-08	2.8181E-08	7.23E-08	7.23E-08	1.15E-07	1.15E-07
10	2.364E-34	2.3642E-33	1.62E-28	1.62E-27	4.43E-21	4.43E-20
100	4.08E-295	4.084E-293	5.1E-235	5.1E-233	3.1E-155	3.1E-153
1000	0	0	0	0	0	0
10000	0	0	0	0	0	0

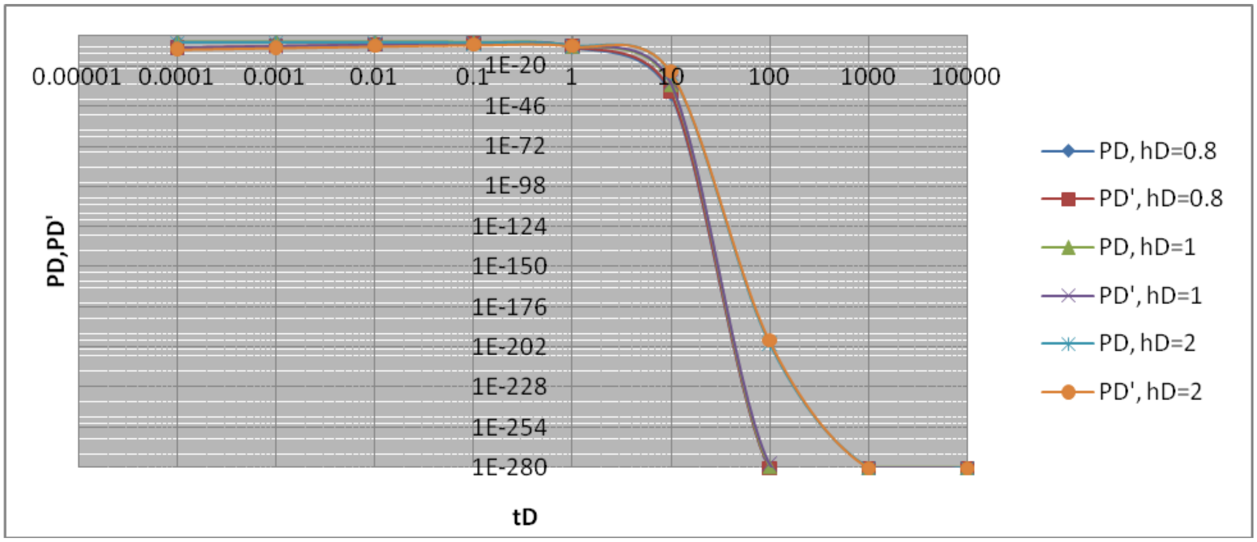

Figure 2: Log-log plot showing the effect of hD on PD and PD' when XeD=4

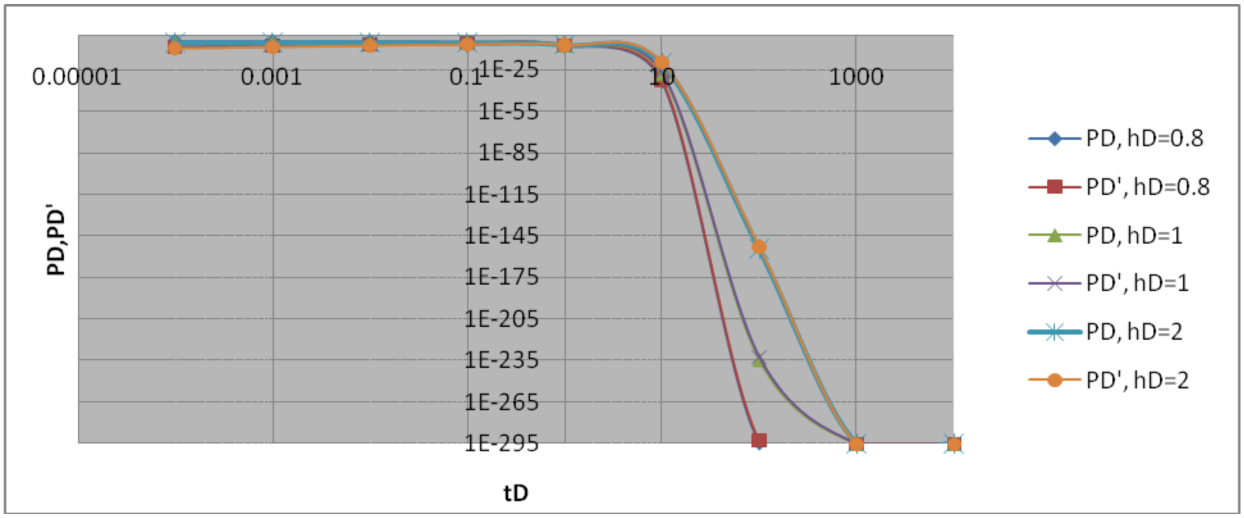

Figure 3: Log-log plot showing the effect of hD on PD and PD' when XeD=8

The Figures 2 and 3 shows that, at early time (pseudo-steady state) at higher value of dimensionless pay thickness (hD), the dimensionless pressures increases and when approaching the late period, a combine pressure effect become noticeable. At late period (steady state) that is beyond tD=1, it was observed that they all felt constant pressure boundary on time except the dimensionless pressure with higher dimensionless pay thickness which exhibit a level of pseudo steady state at late period.

3.1.3 Effect of Reservoir Length on Dimensionless Pressures and Derivative

The effect of dimensionless lateral extent on dimensionless pressure and derivative was investigated, keeping dimensionless wellbore and reservoir parameters constant with different values of dimensionless lateral extent (XeD=4, 8 and 15) were investigated for their effect on dimensionless pressures and derivative. The results are tabulated in table 3 and Figure 4.

Table 3: Dimensionless Pressures and derivative results for varying external lateral extent, XeD

tD	PD, XeD=4	PD', XeD=4	PD, XeD=8	PD', XeD=8	PD, XeD=15	PD', XeD=15
0.0001	4.4464E-05	4.4464E-09	2.223E-05	2.223E-09	1.1857E-05	1.18566E-09
0.001	4.4156E-05	4.4156E-08	2.21E-05	2.21E-08	1.1788E-05	1.17883E-08
0.01	4.1195E-05	4.1195E-07	2.081E-05	2.081E-07	1.1126E-05	1.11262E-07
0.1	2.0577E-05	2.0577E-06	1.142E-05	1.142E-06	6.2415E-06	6.24151E-07
1	1.9897E-08	1.9897E-08	2.818E-08	2.818E-08	1.9263E-08	1.92634E-08

10	1.4217E-38	1.4217E-37	2.364E-34	2.364E-33	1.5107E-33	1.51068E-32
100	0	0	4.08E-295	4.08E-293	1.329E-284	1.3291E-282
1000	0	0	0	0	0	0
10000	0	0	0	0	0	0

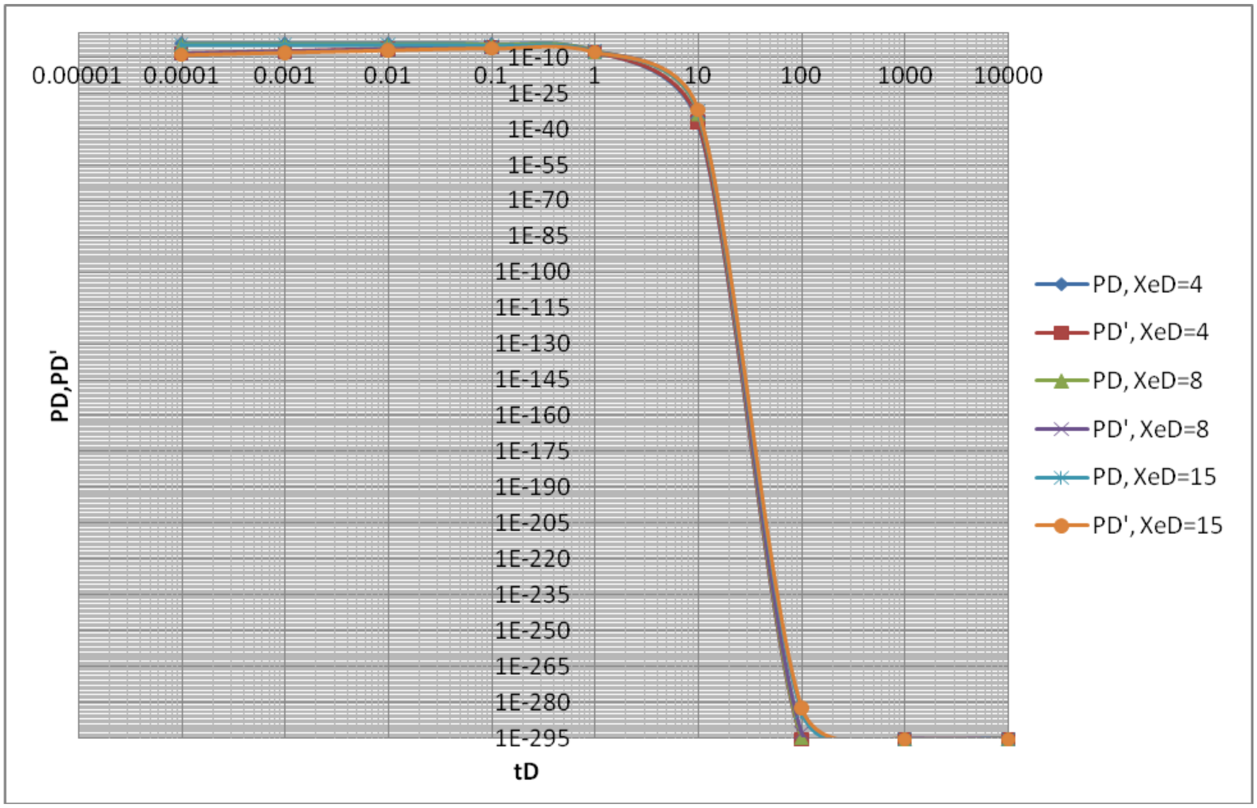

Figure 4: Log-log plot showing the effect of XeD on PD and PD'

In Figure4, it was observed that different dimensionless lateral extent has minimal impact on dimensionless pressure and derivative owing to the fact that other parameters are held constant thereby creating a narrow path along the reservoir in the x-direction.

3.1. 4 Effect of Distance between the Reservoir Bottom and the Wellbore on Dimensionless Pressures and Derivative

The effect of dimensionless distance from the bottom of the reservoir to the wellbore, ZwD on dimensionless pressure and derivative was investigated, keeping dimensionless wellbore and reservoir parameters constant with different values of dimensionless distance from the bottom of the reservoir to the wellbore, (ZwD=0.64, 0.8, 1 and 2) were investigated for their effect on dimensionless pressures and derivative.

Table 4: Dimensionless Pressures and Derivative results for varying Dimensionless Distance from the bottom of the Reservoir to the wellbore, ZwD

tD	PD, ZwD=0.64	PD', ZwD=0.64	PD, ZwD=0.8	PD', ZwD=0.8	PD, ZwD=1	PD', ZwD=1	PD, ZwD=2	PD', ZwD=2
0.0001	4.44636E-05	4.44636E-09	5.55815E-05	5.55814E-09	6.94667E-05	6.94667E-09	0.000138832	1.38832E-08
0.001	4.41561E-05	4.4156E-08	5.5197E-05	5.5197E-08	6.89862E-05	6.89861E-08	0.000137872	1.37872E-07
0.01	4.1195E-05	4.1195E-07	5.14955E-05	5.14955E-07	6.436E-05	6.436E-07	0.000128626	1.28626E-06
0.1	2.05771E-05	2.05771E-06	2.57223E-05	2.57223E-06	3.21482E-05	3.21482E-06	6.42495E-05	6.42494E-06
1	1.9897E-08	1.9897E-08	2.48721E-08	2.48721E-08	3.10856E-08	3.10856E-08	6.21259E-08	6.21259E-08
10	1.42174E-38	1.42174E-37	1.77724E-38	1.77724E-37	2.22123E-38	2.22123E-37	4.43922E-38	4.43921E-37
100	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	0	0	0
10000	0	0	0	0	0	0	0	0

In Table 4, it was observed that dimensionless distance from the bottom of the reservoir to the wellbore has effect on dimensionless pressures and derivative.

3.2 Discussion of Results

Dimensionless pressure and dimensionless pressure derivatives were computed for the various reservoir and well parameters. Results from Table 1 show that, the higher the value of well length, the smaller the dimensionless pressure and also the magnitude of dimensionless pressures derivative increases at early time with increasing well length and decreases as it approaches late time which indicates that longer well lengths will produce more crude in less time while shorterwell lengths will prolong production.Results from figures 2 and 3 indicates that, the higher the pay thickness and the external lateral extent of the reservoir, the more the external fluid encroachment toward wellbore is delayed.

Results from Table 4 implies that, the higher the distance from the bottom of the reservoir the larger the dimensionless pressure and derivative and hence the better the productivity of the well and reservoir.

Conclusion

A model for dimensionless pressure and dimensionless pressure derivative distribution of a horizontal well subjected to a single edge and bottom drive mechanism in a homogeneous isotropic reservoir is presented. Type curves were also generated for the analysis of a horizontal well subjected to a single edge and bottom drive mechanism. The effect of reservoir and well characteristics on the distribution of dimensionless pressure and dimensionless pressure derivatives of a horizontal well in a constrained isotropic reservoir with constant pressure at the bottom and edge has been investigated. It is possible to draw the following conclusions:

1. Shorter well lengths have more effect on dimensionless pressure while longer well length have more effect on dimensionless pressure derivative at the early time, but as it approaches late time thedimensionless pressure derivative decreases until it reaches zero. Therefore longer well lengths will produce more crude in less time while short well lengths will prolong production before external fluid encroachment toward wellbore.
2. Higher pay thickness and external lateral extent of the reservoir are more productive and capable of delaying the encroachment of external fluid toward wellbore.
3. Dimensionless external lateral extent also increases productivity. But for very short well lengths and pay thickness, the effect of dimensionless external lateral extent become insignificant at late time because of the presence of constant pressure boundaries.
4. Higher dimensionless distance from the bottom of the reservoir to the wellbore, ZwD, increases the productivity of the system as the wellbore is located in a position far away from the constant pressure boundaries.
5. Distance from the constant pressure boundaries, well length and pay thickness are the parameters that determine time to reach steady state.

Finally, this study will serve as guidelines to oil producers in well design and appropriate well positioning for efficient withdrawal of crude in a reservoir subjected to single edge and bottom water drive.

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Nomenclature

B	*oil volumetric factor, bbl/STB*
c_t	*total compressibility, 1/psi*
h	*formation thickness, ft*
k	*permeability, md*
L	*total length of horizontal well, ft*
Δp	*pressure drop, psi*
h_D	*dimensionless reservoir height or pay thickness*
L_D	*dimensionless well length*
n	*number of image wells*
P_D	*dimensionless pressure*
P_D'	*dimensionless pressure derivative*
t_D	*dimensionless time*
X_{eD}	*dimensionless reservoir length*
X_D	*perforated length*
Y_{eD}	*dimensionless reservoir width*
Z_D	*the dimensionless distance from the bottom of the reservoir to the center of the wellbore*
Z_{wD}	*the dimensionless distance from the bottom of the reservoir to the bottom of the wellbore*
d_x	*the shortest distance between the well and the x-boundary, ft*
d_y	*the shortest distance between the well and the y-boundary, ft*
d_z	*the shortest distance between the well and the z-boundary, ft*
D_x	*the longest distance between the well and the x-boundary, ft*
D_y	*the longest distance between the well and the y-boundary, ft*
D_z	*the longest distance between the well and the z-boundary, ft*

Greek Symbols

τ	*dummy variable of time*
η	*Diffusivity constant, md-psi/cp*
ϕ	*porosity, fraction*
μ	*viscosity, cp*

Dimensionless Parameters

$$P_D = \frac{kh\Delta p}{141.2q\mu\beta}$$

$$t_D = \frac{4kt}{\phi\mu G L^2}$$

$$\eta_i = \frac{k_i}{\phi\mu G}$$

$$h_D = \frac{2h}{L}$$

$Where i = x, y, z$