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Mathematical Model for Estimating Isothermal Oil Compressibility

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ABSTRACT

The evaluation of petroleum reservoir performance needs a precise understanding of the volumetric behavior of hydrocarbon mixtures, both liquid and gaseous. Prior to evaluating reservoir performance, all transient fluid flow problems require the coefficient of isothermal oil compressibility, which is often determined by reservoir fluid analysis. Sampling and analysis of reservoir fluids is frequently an expensive and time-consuming process that cannot be performed whenever the volumetric attributes of reservoir fluids are required. As a result, engineers rely on correlations designed for the purpose of calculating fluid properties, such as the coefficient of isothermal oil compressibility. A new mathematical model for determining the coefficient of isothermal oil compressibility was established in this study using the Soave Redlich Kwong equation of state (EOS). Four case studies were utilized to demonstrate that the new coefficient of isothermal oil compressibility closely matches experimental values and has the lowest average absolute relative error. Therefore, the new mathematical model can be used for quick prediction of C_o in the absent of laboratory PVT data.

1. Introduction

PVT analysis is typically performed in the laboratory to determine the phase behavior and composition of vapour and liquid in oil and gas reservoirs as a function of pressure, volume, and temperature. PVT Analysis is performed to collect data for the purpose of calculating the isothermal compressibility. To account for gas escaping the solution, oil isothermal compressibility can be defined using oil and gas properties. In practice, isothermal compressibility of oil is determined by sampling reservoir fluid above bubble point pressure, which is typically time consuming and not frequently available due to the associated cost. As a result, various

authors have devised alternative correlations for calculating the coefficient of isothermal compressibility in the absence of experimental data, which is the topic of this study, which is to develop it from the cubic equation of state.

Additionally, reservoir fluid behavior can be characterized by a state equation, which can be used to accomplish the following objectives. Thus, an equation of state (EOS) is an analytical expression that connects pressure, P, temperature, T, and volume, V. The phrase cubic equation of state refers to an equation that, when extended, has volume terms to the first, second, and third powers.

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while the isothermal compressibility coefficient is defined as the rate of volume change with regard to pressure increase per unit volume while all other variables, including temperature, remain constant. Isothermal compressibility can also be computed via correlations involving the API gravity, the formation volume factor, or the compressibility factor, or by using the compositional material balance equation. Among the PVT correlations currently in use in the petroleum industry, Standing (1974) proposed a graphical correlation for determining the oil compressibility for undersaturated hydrocarbon systems, Whitson and Brule expressed his relationship in mathematical form. Ahmed (1985) used 245 experimental data points to propose a mathematical expression for the isothermal oil compressibility using the gas solubility, using the gas solubility as the only correlating parameter. Other correlating parameter such as oil and gas gravity (γ_o, γ_g) and temperature are implemented in the equation through the gas solubility (R_s). Vasquez and Beggs (1980) developed a correlation for the isothermal oil compressibility with the gas solubility R_s , reservoir temperature T, API° gravity, gas specific gravity γ_g , and reservoir pressure p, Trube (1957) presented a correlation for estimating the pseudo reduced compressibility for natural gases and undersaturated crude oils, the pseudo reduced compressibility was correlated with the pseudo reduced temperature and pressure.

Among the PVT correlations now used in the petroleum industry, Standing (1974) provided a graphical correlation for evaluating the compressibility of oil in undersaturated hydrocarbon systems, while Whitson and Brule described their relationship mathematically. Ahmed (1985)

proposed a mathematical equation for the isothermal oil compressibility utilizing the gas solubility as the only related parameter, based on 245 experimental data points. Additional correlated parameters such as oil and gas gravity (γ_o, γ_g) and temperature are incorporated into the equation via the gas solubility (R_s). Vasquez and Beggs (1980) established a relationship between the isothermal oil compressibility and the gas solubility R_s , the reservoir temperature T, the API° gravity, the gas specific gravity, γ_g , and the reservoir pressure p. Trube (1957) proposed a connection for calculating the pseudo reduced compressibility of natural gases and undersaturated crude oils; the pseudo reduced compressibility was connected with the pseudo reduced temperature and pressure.

Vasquez and Beggs (1980) developed a correlation for the isothermal oil compressibility as a function of gas solubility (R_s), reservoir temperature T, API° gravity, gas specific gravity γ_g and reservoir pressure p. They also proposed the relationship for adjustment of the gas gravity γ_g to the reference separator pressure

$$\gamma_{gs} \quad .$$

Petrosky and Farshad (1993) also proposed a relationship for determining the oil compressibility for undersaturated hydrocarbon systems while Standing (1974) proposed a graphical correlation for determining the oil compressibility for undersaturated hydrocarbon systems. McCain's (1988) study established a correlation for calculating the isothermal compressibility of oil at pressures below the bubble threshold. The discovered correlation produces findings that are accurate to within 10% at pressures greater than 500 psia. Under

500 psia, accuracy is within 20%. He further stated that if the bubble point pressure is known, the accuracy of the correlation can be improved by using the expression proposed by McCain.

The sampling of fluid to determine PVT properties such as the measurement of isothermal compressibility has its limitation due to the fact that experimental data from laboratory analysis are seldom available and cannot be taken at every pressure drop due to costs implication, therefore obtaining an accurate PVT behavior of each reservoir fluid encountered will be costly and time-consuming. Hence, in cases when the experimental data are not available, PVT properties such as the isothermal oil compressibility are determined from empirically derived correlations or equation of state. Each of the developed PVT correlations is only applicable to a good degree of reliability only in a well-defined range of reservoir fluid characteristics. This is due to the fact that each correlation is developed based on fluid samples from a restricted geological area with similar fluid composition and API gravity.

This study is to develop a new correlation for undersaturated isothermal compressibility, a modification of the Soave Redlich Kwong equation of state, that should reduce the time and cost associated with routine fluid sampling whenever the reservoir's performance is evaluated as its pressure decreases. This will help to provide solutions of transient flow problems in the form of the total isothermal compressibility, well testing analysis applicable in problems of pressure buildup and drawdown, material balance equation for undersaturated oil reservoirs, to estimate the initial reserve and predict future production and modeling well performance (inflow performance relationship) under

transient flow conditions for both single phase and two-phase reservoir.

1. Methodology

The new mathematical model for the determination of coefficient of oil isothermal compressibility is derived from cubic equation of state. A comparative study was performed on the some of the existing equation of state (EOS) such as Soave Redlich Kwong and Peng Robinson equation

of state. with the new equation developed in this study which is basically a modification of

Soave Redlich Kwong EOS. Furthermore, the new equation developed in this study was validated with a typical field data.

1.1. Derivation of Mathematical Model (New Model)

The coefficient of isothermal compressibility is expressed by

$$c_o = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T \quad (1)$$

Soave Redlich Kwong Equation of State relating pressure, Temperature and volume (PVT) is expressed by

$$p = \frac{RT}{V-b} - \frac{a}{V(V+b)} \quad (2)$$

Differentiating with respect to volume and pressure keeping temperature constant yield the following using quotient rule. The term $\frac{RT}{V-b}$ becomes

$$\frac{\partial}{\partial v} \left(\frac{RT}{V-b} \right)_T = \frac{(V-b) \frac{\partial}{\partial v} (RT) - (RT) \frac{\partial}{\partial v} (V-b)}{(V-b)^2} \quad (3)$$

$$\frac{\partial}{\partial v} \left(\frac{RT}{V-b} \right)_T = \frac{-RT}{(V-b)^2} \quad (4)$$

The term $\frac{a}{v(v+b)}$ become

$$\frac{\partial}{\partial v} \left(\frac{a}{v(v+b)} \right)_T = \frac{(v^2-vb) \frac{\partial}{\partial v}(a) - (a) \frac{\partial}{\partial v}(v^2-vb)}{(v^2-vb)^2}$$

$$\frac{\partial}{\partial v} \left(\frac{a}{v(v+b)} \right)_T = - \frac{a(2v+b)}{(v^2-vb)^2}$$

(5)

The term $\left(\frac{\partial P}{\partial v} \right)_T$

becomes

$$\left(\frac{\partial p}{\partial v} \right)_T = \frac{a(2v+b)(v-b)^2 - RT(v^2-vb)}{(v-b)^2(v^2-vb)}$$

(6)

The term $\left(\frac{\partial v}{\partial p} \right)_T$

becomes

$$\frac{(v-b)^2(v^2+vb)^2}{a(2v+b)(v-b)^2 - RT(v^2+vb)^2}$$

(7)

Hence, the new oil isothermal compressibility for this study becomes

$$C_o = \left(-\frac{1}{v} \right) \frac{(v-b)^2(v^2+vb)^2}{a(2v+b)(v-b)^2 - RT(v^2+vb)^2}$$

(8)

The crude oil system is considered a multi component system and can be expressed by the parameters a_m and b_m represent the mixture parameters for the crude oil system

mixture $C_o =$

$$\left(-\frac{1}{v} \right) \frac{(v-b_m)^2(v^2+vb_m)^2}{a_m(2v+b_m)(v-b_m)^2 - RT(v^2+vb_m)^2}$$

(9)

The molar volume (V) in the new isothermal oil compressibility equation is calculated via an iterative process using the GRG

minimization function in Microsoft excels. The algorithm for isothermal oil compressibility estimation using cubic equation of state is represented in Figure 1. The model parameter, error analysis and step by step determination of the molar volume from excel are presented in the appendix section.

3. Results and Discussion

The result of the coefficient of oil isothermal compressibility obtained from experimental data is compared to that obtained from the new mathematical model and other existing cubic equation of state (Soave Redlich Kwong and Peng Robinson Equation of state). The compositional data of four wells in the Niger Delta were used as an input data and the result obtained used to validate the new coefficient of oil isothermal compressibility developed in this study at various depletion pressure.

$$\left(\frac{\partial v}{\partial p} \right)_T =$$

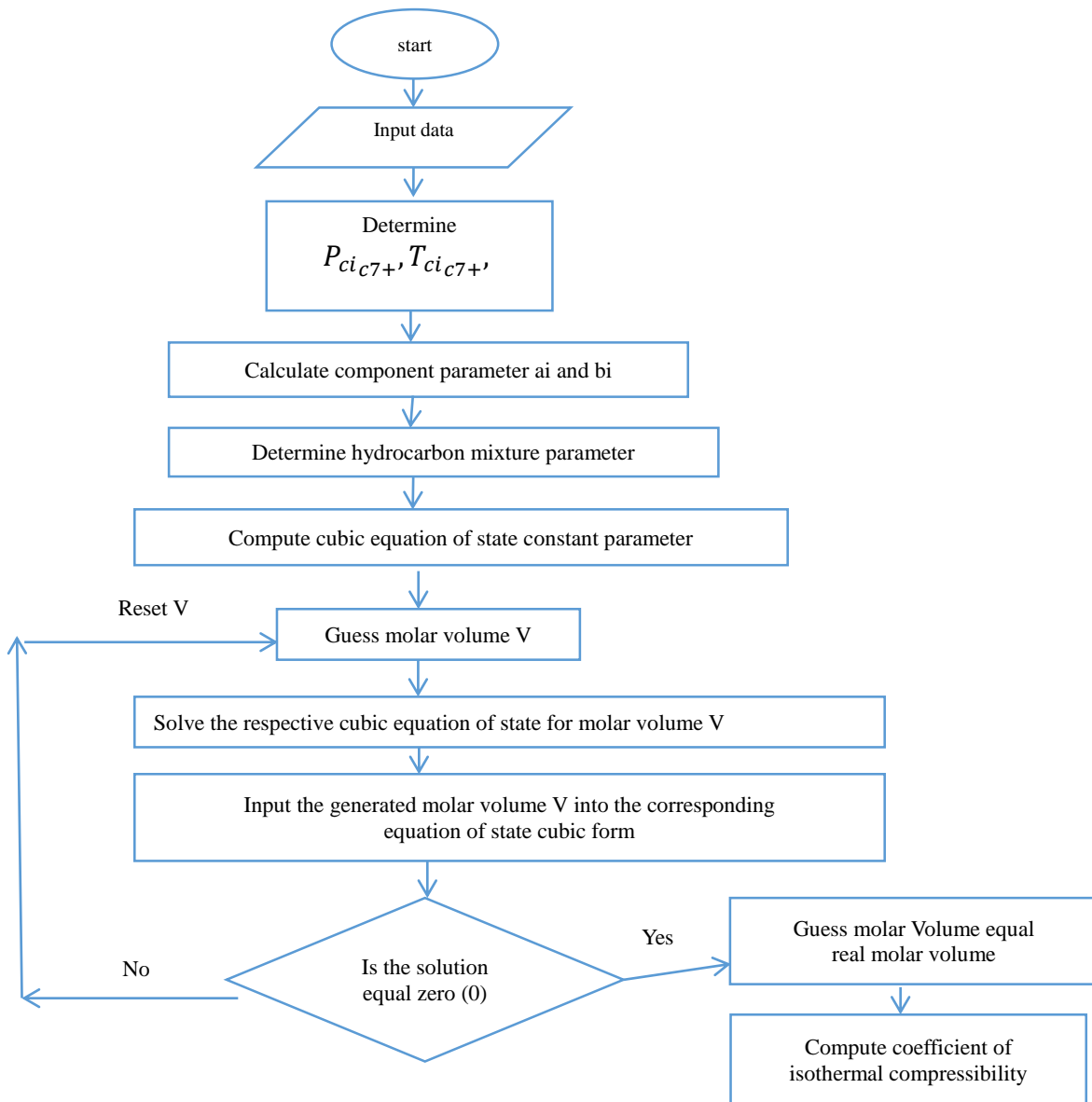


Figure 1: Algorithms for isothermal oil compressibility

Table 1: Compositional Input Data

Well	ASHA #4678	ALPHA #2356	JEKAT A #7655	REHIT A #5556	Critical pressure (psia)	Critical temperature (Rankine)
Crude oil component	Mole fraction	Mole fraction	Mole fraction	Mole fraction		
CO2	0.0025	0.0016	0.0049	0.0091	1070.6	547.6
N2	0.0088	0.0194	0.0053	0.0016	493	227.3
C1	0.2394	0.255	0.3883	0.3647	667.8	343.8
C2	0.1167	0.0733	0.0986	0.0967	707.8	549.8
C3	0.0936	0.1121	0.0953	0.0695	666	617.7
i-C4	0.0139	0.0349	0.0123	0.0144	526.5	734.7
n-C4	0.0461	0.0417	0.0431	0.0393	551	765.7
i-C5	0.015	0.0218	0.012	0.0144	488.85	829.8
n-C5	0.0248	0.0276	0.0187	0.0141	490.4	846.4
C6	0.0326	0.0367	0.0282	0.0433	439.35	914.4
C7+	0.4066	0.3755	0.2933	0.3329		
Molecular weight of C7+	196	209	252	218		
Specific gravity of C7+	0.8494	0.8467	0.8413	0.8515		
Temperature (°F)	247	138	162	220		
bubble point pressure(psig)	1936	1675	1675	2620		

The result of Well ASHA #4678 shows the new model fitting into five experimental data from chart and a little deviation for pressure less than 3500 psia. Peng Robinson derived

correlation shows to a good degree with prediction close to the experimental data; Soave Redlich Kwong was the least accurate of all as shown in Table 2 and Figure 2.

Table 2: Well ASHA #4678 Result

Well ASHA #4678	Compressibility(ψi-1)×10 ⁻⁶				
	Pressure (psia)	Experimental Data	Soave Redlich Kwong EOS	Peng Robinson EOS	This study
	6014.7	10.7	12	11	10.2
	5514.7	11.3	12.7	11.8	11
	5014.7	12	13.8	12.8	11.8
	4514.7	12.7	15	13.9	12.7
	4014.7	13.6	16.4	15.2	13.8
	3514.7	14.6	18.1	16.7	15.1
	3014.7	15.9	20.2	18.5	16.5
	2514.7	17.4	22.7	20.8	18.2
	2014.7	19.5	25.8	23.5	20.3
	1950.7	20.8	26.2	23.9	20.6

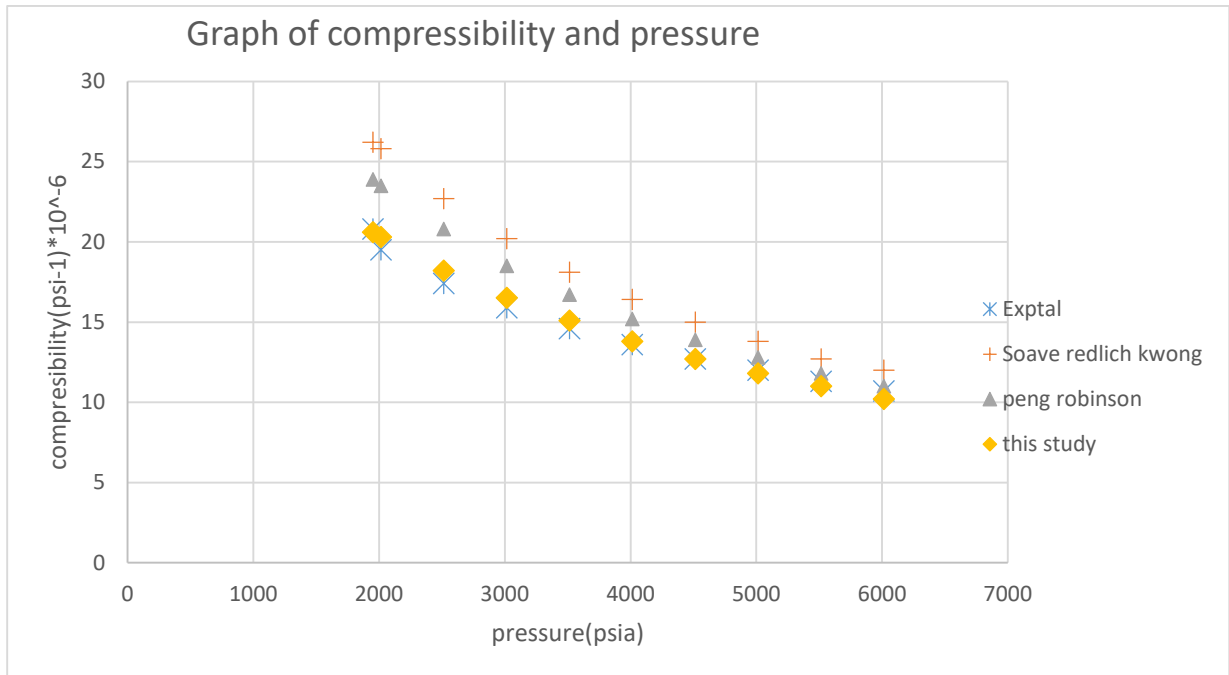


Figure 2: Well ASHA #4678 isothermal compressibility chart

The result of Well ALPHA #2356 shows also a close estimation to experimental values than Peng Robinson and Soave Redlich Kwong derived compressibility (Figure 3). A

significant deviation of the new model from experimental was notice at pressure below 3000psia.

Table 3: Well ALPHA #2356 RESULT

WELL ALPHA #2356	Compressibility(psi-1) ×10 ⁻⁶			
	Experimental Data	Soave Redlich Kwong EOS	Peng Robinson EOS	This study
4514.7	8.46	9.86	8.87	8.29
4014.7	8.83	10.6	9.54	8.86
3514.7	9.41	11.5	10.3	9.51
3014.7	9.97	12.6	11.2	10.2
2514.7	10.7	13.8	12.2	11.1
2014.7	11.5	15.2	13.4	12
1914.7	12	15.5	13.6	12.2
1814.7	12.01	15.8	13.9	12.5

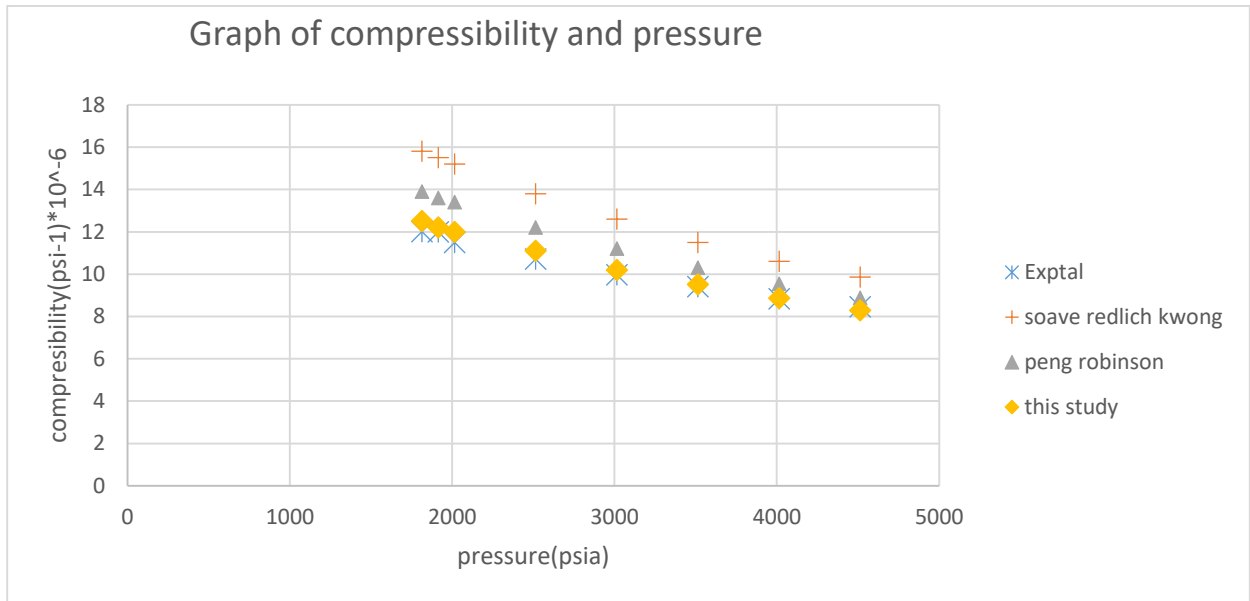


Figure 3: Well ALPHA #2356 Isothermal Compressibility Chart

The result of Well JEKATA #7655 also exhibits a similar trend to that of Well ASHA #4678 and also Well ALPHA #2356 with the new model showing a close approximation to experimental data than Peng Robinson and

Soave Redlich Kwong derived compressibility, although the new model notices an earlier deviation from experimental data (Figure 4).

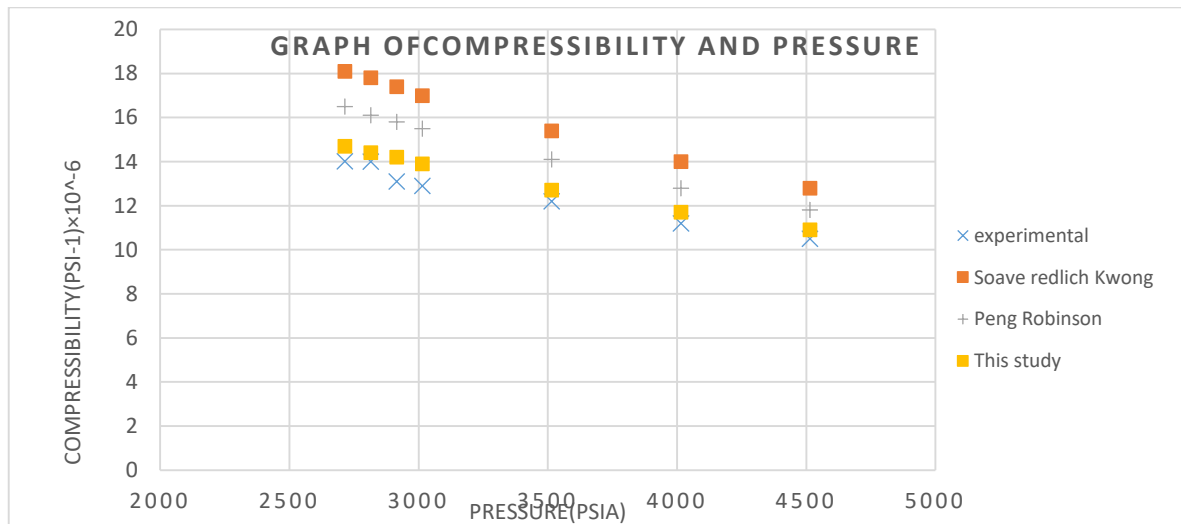


Figure 4: Well JEKATA #7655 Isothermal Compressibility Chart

The result of Well REHITA #5556 shows also a close approximation to experimental data. At pressure less than 2750psia the new model exhibit compressibility values lower than

experimental values. In all the new performs better than Peng Robinson and Soave Redlich Kwong derived compressibility (Figure 5).

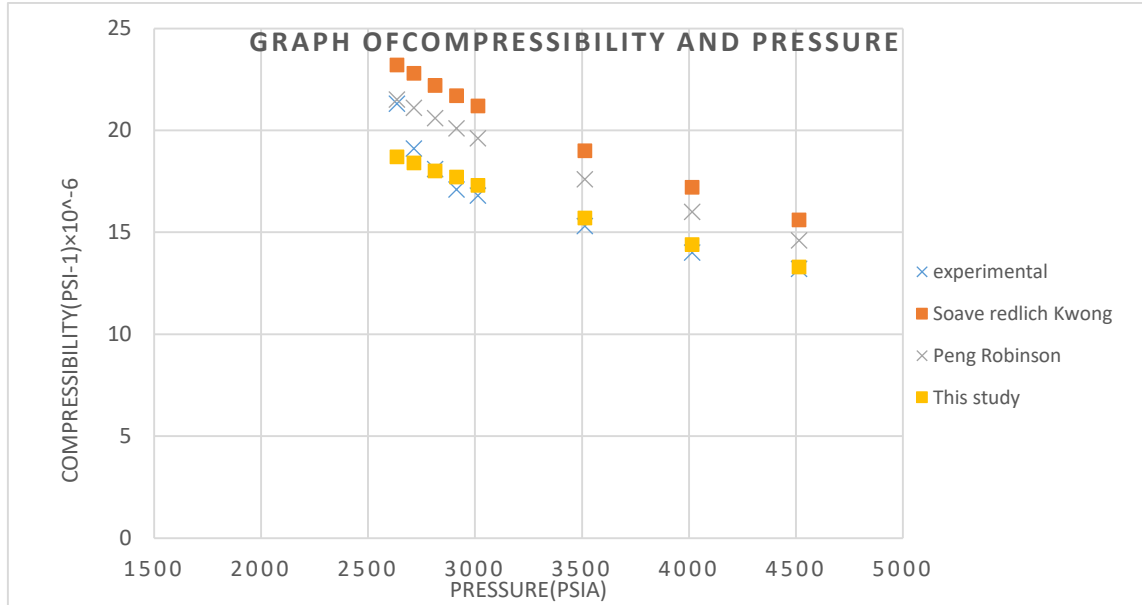


Figure 5: Well REHITA #5556 Isothermal Compressibility Chart

3.1 Error Analysis

Error analysis was done to ascertain the accuracy of the new model developed in this study. Data from well ASHA #4678, WELL ALPHA #2356, WELL JEKATA #7655 and WELL REHITA #5556 were used, and the results are shown respectively.

Result from error analysis using statistical method on the new mathematical model yield mean absolute percent error in the range of 2.45 and 5.19 and standard deviation in the range of 3.77 and 1.47 and thus, agree with literature that the best correlation should have the least Average

Absolute Relative Error, AARE and lowest Standard Deviation, SD. and Relative Standard Deviation (RSD).

Table 4: Well ASHA #4678 error analysis

Statistical Parameters	Soave Redlich Kwong EOS	Peng Robinson EOS	This study
Average Absolute Relative Error, AARE	21.8	12.1	2.75
Standard Deviation, S	28.7	28	25.1
Relative Standard Deviation (RSD)	5.24	4.71	3.77

Table 5: Well ALPHA #2356 error analysis

Statistical Parameters	Soave Redlich Kwong EOS	Peng Robinson EOS	This study
Average Absolute Relative Error, AARE	25.9	11.8	2.45
Standard Deviation, S	17.7	16.2	15.2
Relative Standard Deviation (RSD)	2.51	1.94	1.61

Table 6 Well JEKATA #7655 error analysis

Statistical Parameters	Soave Redlich Kwong EOS	Peng Robinson EOS	This Study
Average Absolute Relative Error, AARE	27.7	16.5	5.19
Standard Deviation, S	12.8	12.3	11.1
Relative Standard Deviation (RSD)	2.05	1.8	1.47

Table 7 Well REHITA #5556 error analysis

Statistical Parameters	Soave Redlich Kwong EOS	Peng Robinson EOS	This Study
Average Absolute Relative Error, AARE	21.2	12.4	3.62
Standard Deviation, S	13.7	13.4	11.9
Relative Standard Deviation (RSD)	2.79	2.54	1.99

Conclusion

The following conclusions can be drawn from this project:

- i. A mathematical model based on Soave Redlich Kwong and Peng-Robinson equation of state for calculating the Coefficient of isothermal oil compressibility was developed.
- ii. The mathematical model was extended to pressures above the bubble point
- iii. The new model performs better than every other correlation when validated with field data
- iv. A good degree of accuracy was obtained between the predicted coefficient of isothermal oil compressibility from the developed EOS based mathematical model and the experimentally derived coefficient of isothermal compressibility.

List of Abbreviations

Abbreviation	Meaning
EOS	Equation of state

PVT	Pressure-volume-temperature
API	America Petroleum Institute
AARE	Average absolute relative error
SD	Standard deviation
RSD	Relative standard deviation

Availability of data and materials

The data and materials for this study are available in the manuscript submitted.

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Appendix

Constant parameters used in predicting oil isothermal compressibility are given in Table 3.1.

Table 3.1: Model parameters

	Soave Redlich Kwong EOS	Peng Robinson EOS	This Study
Parameter a	0.42747	0.45724	0.42747
Parameter b	0.08664	0.0778	0.0796

Step by Step Calculation Process Using Excel Software

GETTING STARTED

- Open excel, go to **FILE>NEW** to create a fresh Microsoft excel page.
- go to **FILE>SAVE AS** to save the fresh excel page created

- starting from the top upright of the excel page, click on cell A1 name cell A1 as COMPONENT enter the name of all the component present in the hydrocarbon mixture from cell A2
- click on cell B1 name cell B1 as mole fraction enter the corresponding mole fraction with respect to the component name in column A.
- click on cell C1 name cell C1 as Critical Temperature enter the critical temperature values with respect to the component name in column A as in table 4.1
- click on cell D1 name cell D1 as Critical Pressure enter the critical pressure values with respect to the component name in column A as in table 4.1
- create a column (e.g. column E) for acentric factor and enter values with respect to the component name in column A
- create a Row (e.g. Row 20) and enter values of the reservoir Temperature, Pressure, Molecular weight of heptane plus and specific gravity of the heptane plus fraction, name the corresponding value using row 19.
- Determine the critical properties of the heptane plus fraction using a correlation by Lawal-Tododo-Heinze and enter the values in the respective cell with relation to the component name in column A using equation 3.18 to 3.22
- click on cell F1 name cell F1 as Temperature correction parameter $\alpha_i(T)$ and determine the Temperature correction parameter $\alpha_i(T)$ for each component I using the right equation(equ 2.20, 2.21, 2.28 and 2.29)
- click on cell G1 name cell G1 as Attraction parameter a_i and determine the parameter a_i for each component with respect to the critical properties for a particular cubic equation of state
- click on cell H1 name cell H1 as Repulsion parameter b and determine the parameter b for each component with

respect to the critical properties for a particular cubic equation of state

- Determine the hydrocarbon mixture parameter a_m and b_m from the computed attraction and repulsion parameter a_i and b_i using the mixing rules and enter values into a blank cell at row 20 and the corresponding name in row 19
- Determine the constant term in the respective equation of state define as parameter A, B and C in cell I1 to I3

For soave Redlich Kwong EOS

$$A = \left(\frac{RT}{P}\right)$$

$$B = \left(\frac{a_m}{p} - \frac{b_m RT}{p} - b_m^2\right)$$

$$C = \frac{a_m b_m}{p}$$

For Peng Robinson EOS

$$A = \left(b_m - \frac{RT}{p}\right)$$

$$B = \left(\frac{a_m}{p} - 3b_m^2 - 2b_m \frac{RT}{p}\right)$$

$$C = \left(b_m^3 + \frac{RT}{p} b_m^2 - \frac{a_m b_m}{p}\right)$$

- Enter an initial guess for molar volume into cell
- Solve the respective Equation of state for the molar volume using excel solver to obtain the term V

For soave Redlich Kwong EOS

$$V^3 - AV^2 + BV - C = 0$$

For Peng Robinson EOS

