Research Article / Review Article FUPRE JOURNAL 8(1): 01-10(2024)

FUPRE Journal

of

Scientific and Industrial Research

ISSN: 2579-1184(Print) ISSN: 2578-1129 (Online)

http://fupre.edu.ng/journal

Theoretical Analysis of the Exponentiated Alpha Power Inverted Exponential Distribution: Properties and Application

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ARTICLE INFO

Received: 01/07/2023

Accepted: 29/11/2023

Keywords

Alpha power inverted exponential, Alpha power transformation, Breast cancer, Life time Distribution, Maximum Likelihood.

ABSTRACT

The Exponentiated Alpha Power Inverted Exponential (E-APIE) distribution is studied. Breast cancer data and glass fiber data were used to examine the model competence. The failure rate, cumulative, survival, order statistics, odd functions, reversed hazard were investigated. Maximum likelihood procedure was employed to acquire the estimated parameters. A simulation was conducted to examine the estimator's performance and relevance. The outcomes of the simulation and real-life analysis indicated that the E-APIE distribution produced a better result.

1. INTRODUCTION

The Alpha Power Transformation (APT) proposed in Mahdavi and Kundu (2017) has received considerable attentions in recent time. This transformation allows an extra parameter to be added to the family of distributions. The extra parameter makes the class of distributions more flexible. However, Mahdavi and Kundu (2017) adopted the APT technique in transforming the exponential distribution and obtained a new distribution called the alpha power exponential (APE) distribution. This transformation was further generalized in Dey et al. (2017a). However, Dey et al. (2017b) extended this transformation to Weibull distribution. Eghwerido et al. (2019) and Efe-Eyefia et al. (2019) applied the Gompertz-G and Weibull-G transformation technique respectively in developing a more flexible alpha power inverted exponential distribution. Malik and Ahmad (2019) proposed the transmuted alpha power inverse Rayleigh distribution. Eghwerido et al. (2021) proposed the alpha power Gompertz distribution. Alzaghal et al. (2013), Aljarrah et al. (2014), and Alzaatreh et al. (2013) proposed the exponentiated T-X family of distributions and a method for generating family of distributions. Aryal et al. (2017) proposed the exponentiated generalized-G Poisson family of distributions. Transmuted Topp-Leone-G, Burr-X and Marshall-Olkin generalized-G family of distributions were proposed in Yousof et al. (2017, 2017a) and Yousof et al. (2018).

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Thus, for a random variable W, Unal et al. (2018) proposed the alpha power inverted exponential (APIE) with the probability density function given as

$$f(w)=\frac{\beta\log\alpha}{w^{2}(\alpha-1)}\exp\left(-\frac{\beta}{w}\right)\alpha^{\exp\left(-\frac{\beta}{w}\right)},\alpha\neq1,\alpha>0,w>0 \quad (1)$$

The corresponding cumulative distribution function is given as

$$F(x)=(\alpha-1)^{-1}\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right),\ for\ w>0,\alpha>0,\ \alpha\neq1 \quad (2)$$

However, despite the transformation of [8], the inverted exponential distribution was not flexible in terms of the test statistic and p-value of its estimated parameters. Hence, this study is motivated by adding extra parameter to make the new class of distribution more flexible than APIE distribution.

This study aimed at proposing a three-parameter class of the alpha power inverted exponential distributions together with some of its basic statistical properties. It further examined its efficiency by applying to real life data.

2. THE E-APIE DISTRIBUTION

Let $w_1, w_2, \ldots, w_n$ be a random sample with a baseline cdf $G(w)$ and pdf $g(w)$. Then the cdf of the exponentiated APT (E-APT) distribution is given as

$$F(w)=(\alpha-1)^{-a}\left(\alpha^{G(w)}-1\right)^{a} \quad \alpha\neq1,\ \alpha>0,a>1. \quad (3)$$

The pdf of the exponentiated APT (E-APT) distribution is given as

$$f(w)=\frac{ag(w)\log\alpha}{(\alpha-1)^{a}}\alpha^{G(w)}\left(\alpha^{G(w)}-1\right)^{a-1}\ \alpha\neq1,a>1 \quad (4)$$

where a is the additional exponentiated parameter.

The Equation (3) and Equation (4) can be expressed as a new three parameter form with cdf and pdf of the proposed E-APIE model give as

$$F(w)=\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^{a}(\alpha-1)^{-a},\alpha\neq1,a>1,\beta>0. \quad (5)$$

$$f(w)=\frac{a\beta\exp\left(-\frac{\beta}{w}\right)\log\alpha}{(\alpha-1)^{a}w^{2}}\alpha^{\exp\left(-\frac{\beta}{w}\right)}\times\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^{a-1},\quad \alpha\neq1,\beta>0,\quad a>1, \quad (6)$$

where β is the scale parameter and α is the shape parameter.

The following can be derived from the proposed density function.

- If $a=1$, we obtain the alpha power inverted exponential distribution.
- If $a=1$, $\alpha=1$, we obtain the inverted exponential distribution.

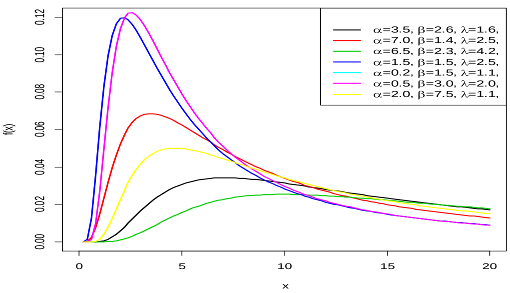

Figure 1: The pdf of the E-APIE distribution with different parameter values

Figure 1 shows the pdf of the E-APIE distribution for various parameter values. In Figure 1, the shape of the E-APIE distribution could be inverted, unimodal and positively skewed.

3. MAIN STATISTICAL PROPERTIES OF THE E-APIE DISTRIBUTION

This section investigates some statistical properties of the E-APIE distribution.

3.1 Quantile function and random number generation

Let W be a random variable such that $W \sim E-APIE(a,\beta,\alpha)$. Then, the quantile function of W. For $u \in (0,1)$ is given as

$$w_u = -\beta \log\left\{(\log\alpha)^{-1}\log\left[(\alpha-1)u^{\frac{1}{a}}+1\right]\right\}^{-1},\quad 0<u<1. \tag{7}$$

In particular, the median (M) of the random variable W is obtained as

$$M = -\beta \log\left\{(\log\alpha)^{-1}\log\left[(\alpha-1)0.5^{\frac{1}{a}}+1\right]\right\}^{-1} \tag{8}$$

However, the 25^{th} and 75^{th} percentile are obtained respectively as

$$Q_1 = -\beta \log\left\{(\log\alpha)^{-1}\log\left[(\alpha-1)0.25^{\frac{1}{a}}+1\right]\right\}^{-1} \tag{9}$$

$$Q_3 = -\beta \log\left\{(\log\alpha)^{-1}\log\left[(\alpha-1)0.75^{\frac{1}{a}}+1\right]\right\}^{-1} \tag{10}$$

The E-APIE random variable can be simulated straightforward. If u is a uniform variate on the unit interval $(0,1)$.Then, the random variable $W = w_u$ follows Equation (6).

3.2 The distribution of the E-APIE order statistics

Suppose $w_1, w_2, \ldots, w_n$ are random samples drawn from an infinite population with a pdf of the E-APIE distribution. Then the pdf of the k^{th} order statistics is given as

$$g_k(w) = \frac{n!}{k(k-1)!(n-k)(n-k-1)!}\left[\frac{\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1}{\alpha-1}\right]^{a(k-1)} \times\left[\frac{(\alpha-1)^a-\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^a}{(\alpha-1)^a}\right]^{n-k}\frac{a\beta\exp\left(-\frac{\beta}{w}\right)\log\alpha}{(\alpha-1)^a w^2} \times\alpha^{\exp\left(-\frac{\beta}{w}\right)}\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^{a-1} \tag{11}$$

The minimum and maximum order statistics are obtained by setting $k=1$ and $k=n$ respectively.

3.3 The odds function

The odds function of the E-APIE distribution is given as

$$O(x) = \frac{\left[\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right]^a(\alpha-1)^a}{1-\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^a(\alpha-1)^a} \tag{12}$$

3.4 Cumulative hazard rate function

The cumulative hazard function of the E-APIE distribution is given as

$$H(x) = -In\left\{1-\left[\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right]^{a}(\alpha-1)^{a}\right\}$$

(13)

3.5 Reserved hazard rate function

The reversed hazard function of the E-APIE distribution is given as

$$r(x) = \frac{\frac{a\beta\exp\left(-\frac{\beta}{w}\right)\log\alpha}{(\alpha-1)^{a}w^{2}}\alpha^{\exp\left(-\frac{\beta}{w}\right)}\left(\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1\right)^{a-1}}{\left[\frac{\alpha^{\exp\left(-\frac{\beta}{w}\right)}-1}{\alpha-1}\right]^{a}}.$$

(14)

1. Parameter estimation for the exponentiated alpha power inverted exponential distribution formulation.

4. PARAMETER ESTIMATION FOR THE E-APIE DISTRIBUTION FORMULATION

Let $w_1, w_{2,}, \cdots, w_n$ be random variable obtained from a population with weilbull alpha power inverted exponential distribution. Then, the log-likelihood of exponential alpha power inverted exponential distribution for vector $\Theta = (a, \alpha, \beta)^T$ can be represented as $\ell_n(w_i, \Theta)$ as

$$\ell_n(w_i,\Theta) = n\log a + n\log\beta - \sum_{i=1}^{n}\frac{\beta}{w_i} + n\log(\log\alpha) + n\exp\left(-\frac{\beta}{w_i}\right) + n\exp\left(-\frac{\beta}{w_i}\right)\log\alpha - an\log(\alpha-1) - \sum_{i=1}^{n}\log w_i^2(a-1) \times\log\left(\alpha^{\exp\left(-\sum_{i=1}^{n}\frac{\beta}{w}\right)}-1\right).$$

(15)

Taking partial derivative of Equation (15) with respect to the parameters (a, α, β) and equating to zero. The solution to the nonlinear equation for the parameters can be obtained using R software, MATLAB, and MAPLE. Thus, yield the maximum likelihood estimate $\hat{\Theta} = (\hat{a}, \hat{\beta}, \hat{\alpha})$.

5. SIMULATION STUDY

A simulation is carried out to test the flexibility and efficiency of the E-APIE distribution. Table 1 shows the simulation for different values of parameters for the E-APIE distribution. The simulation is performed as follows:

Data are generated using

$$x_u = -\beta\log\left\{(\log\alpha)^{-1}\log\left[(\alpha-1)u^{\frac{1}{a}}+1\right]\right\}^{-1}, 0 < u < 1$$

.

The values of the parameters are set as $I: \alpha = 1.3,\ \beta = 0.5,\ \theta = 2.0$ and $II: \alpha = 3.0,\ \beta = 0.7,\ \theta = 1.5$.

The sample sizes are taken as n = 50, 75, 100, 150, 200 and 300.

Each sample size is replicated 1000 times.

In this simulation study, we investigated the mean estimates (MEs), variance, biases and root means squared errors (RMSEs) of the maximum likelihood estimate (MLEs).

The bias is calculated by $for\ (S = a, \lambda, \alpha)$

$$\hat{Bias} = \frac{1}{1000}\sum_{i=1}^{1000}\left(\hat{S}_i - S\right)$$

Also, the MSE is obtained as

$$\hat{MSE} = \frac{1}{1000}\sum_{i=1}^{1000}\left(\hat{S}_i - S\right)^2 .$$

Table 1 shows the results of the Monte Carlo study. The results in Table 1 show that the average estimates, biases and RMSEs of the MLEs of the parameters. The RMSEs and the biases decrease as the sample size increases and approach zero. This corresponds to the first-order asymptotic theory. The mean estimates of the parameters approach the true parameter values as the sample size increases. The variance decreases in all the cases as the sample size increases.

Table 1: Simulation Results: - Mean, Bias and RMSE *1,000* MLEs of E-APIE distribution for some fixed parameter values

I				
n	*Parameter.*	*Mean*	*Bias*	*RMSE*
	$\alpha = 1.3$	2.1265	0.8265	0.9005
50	$\beta = 0.5$	-1.485	-1.985	2.0251
	$a = 2.0$	2.5003	0.5003	0.5343
	$\alpha = 1.3$	2.0827	0.7827	0.8562
75	$\beta = 0.5$	-1.365	-1.865	1.8931
	$a = 2.0$	2.4873	0.4873	0.5181
	$\alpha = 1.3$	2.0558	0.7558	0.8244
100	$\beta = 0.5$	-1.283	-1.783	1.8044
	$a = 2.0$	2.4820	0.4820	0.5129
	$\alpha = 1.3$	2.0089	0.7089	0.7750
150	$\beta = 0.5$	-1.181	-1.681	1.6983
	$a = 2.0$	2.4619	0.4619	0.4861
	$\alpha = 1.3$	1.9506	0.6506	0.6985
200	$\beta = 0.5$	-1.124	-1.624	1.6394
	$a = 2.0$	2.4453	0.4453	0.4647
	$\alpha = 1.3$	1.9357	0.6357	0.6861
300	$\beta = 0.5$	-1.047	-1.547	1.5607
	$a = 2.0$	2.4403	0.4403	0.4649

II				
n	*Parameter.*	*Mean*	*Biases*	*RMSE*
	$\alpha = 3.0$	3.5758	-	0.6568
50	$\beta = 0.7$	-1.689	0.5758	2.4430
	$a = 1.5$	2.3615	-	0.9016
			2.3898	
			0.8615	
	$\alpha = 3.0$	3.5411	0.5411	0.6363
75	$\beta = 0.7$	-1.533	-	2.2707
	$a = 1.5$	2.3281	2.2335	0.8717
			0.8281	
	$\alpha = 3.0$	3.5010	0.5010	0.6195
100	$\beta = 0.7$	-1.429	-	2.1591
	$a = 1.5$	2.3141	2.1298	0.8555
			0.8141	
	$\alpha = 3.0$	3.4441	0.4441	0.5891
150	$\beta = 0.7$	-1.297	-	2.0209
	$a = 1.5$	2.3053	1.9973	0.8560
			0.8053	
	$\alpha = 3.0$	3.4151	0.4151	0.5756
200	$\beta = 0.7$	-1.216	-	1.9352
	$a = 1.5$	2.3064	1.9163	0.8614
			0.8064	
	$\alpha = 3.0$	3.3796	0.3796	0.5623
300	$\beta = 0.7$	-1.121	-	1.8372
	$a = 1.5$	2.3034	1.8219	0.8715
			0.8034	

6. APPLICATIONS

Breast cancer and glass fiber data were applied to the proposed model to examine the performance of the model based on its statistic. Several criteria were used to determine the distribution for the best fit: Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Bayesian Information Criteria (BIC), and Hannan, Quinn Information Criteria (HQIC), Kolmogorov Smirnov (KS) statistic and the p value were provided. The models used for comparison are transmuted Alpha Power Inverse Rayleigh Distribution (TAPIRD), Alpha Power Inverse Rayleigh Distribution (APIRD), Transmuted Rayleigh

Distribution TRD and Alpha Power Inverted Exponential Distribution (APIED).

6.1 Breast cancer data

Our second set of data consists of 300 observations taken on cases of breast cancer patients between 2011 and 2016 in University of Ilorin Teaching Hospital (UITH) as used in Oguntunde et al. (2017).

Table 2 shows the measure of comparison for the various distributions under consideration.

Table 2: Performance rating of the EAPIE distribution with breast cancer dataset

Distributions	Parameter MLEs	AIC	CAIC	BIC	HQIC	KS	p-value
E-APIED	$\hat{a}=83.3703$ $\hat{\lambda}=0.1491$ $\hat{\alpha}=35.1187$	2951.617	2951.698	2962.728	2956.064	0.4245	0.1000
TAPIRD	$\hat{a}=6247$ $\hat{\lambda}=0.0905$ $\hat{\alpha}=0.7877$	3988.73	3988.811	3999.841	3993.177	2.8899	0.0001
APIRD	$\hat{\lambda}=132.9441$ $\hat{\alpha}=9.2994$	3005.225	3005.266	3012.633	3008.19	0.4059	0.0001
TRD	$\hat{\lambda}=0.9417$ $\hat{\alpha}=0.3524$	3998.64	3998.68	4006.048	4001.605	0.4109	0.0001
APIED	$\hat{\alpha}=154.0026$ $\hat{\lambda}=8.9206$	3005.504	3005.544	3012.911	3008.468	0.4080	0.0001

6.2 Glass fiber data

The data on 1.5 cm strengths of glass fibres were obtained by workers at the UK National Physical Laboratory was also used to compare the performance of the E-APIE distribution as used in Smith and Naylor (1987), Eghwerido et al (2019), Eghwerido et al (2021a), Eghwerido et al (2021b), Eghwerido et al (2020a), Eghwerido et al (2022a,b,c), Eghwerido and Efe-Eyefia (2022d) Nzei et al. (2020), Eghwerido et al (2020b), Eghwerido et al (2021c), Eghwerido et al (2021d) , Eghwerido et al (2023), Obubu et al. (2019a) and Obubu et al. (2019b).

Table 3 is the measure of comparison for the various distributions under consideration.

6.3 Discussion

The performance of a model is determined by the value that corresponds to the lowest Akaike Information Criteria (AIC). Figures 2, 3, 4 and 5 show the estimated densities and cdfs of the data use. However, the proposed E-APIE model fit the data sets compare to some existing models. In the two real life cases considered, the E-APIE distribution has the lowest AIC value with 185.2713 in glass fibres data and 2951.617 in breast cancer data respectively.

Table 3: Performance rating of the E-APIE distribution with glass fibres dataset

Distributions	Parameter MLEs	AIC	CAIC	BIC	HQIC	KS	p-value
E-APIED	$\hat{a}=2.0122$ $\hat{\lambda}=41.9433$ $\hat{\alpha}=12.5518$	**185.2713**	**185.6781**	**191.7007**	**187.8001**	**0.4873**	**0.71930**
APIED	$\hat{\alpha}=53.5634$ $\hat{\lambda}=0.3509$	196.3253	196.5253	200.611	198.0111	0.4922	2.0764e-12
TAPIRD	$\hat{a}=6247$ $\hat{\lambda}=0.0905$ $\hat{\alpha}=0.7877$	215.5910	215.9968	222.0194	218.1187	0.7306	1.0239e-16
APIRD	$\hat{\lambda}=4557$ $\hat{\alpha}=0.0191$	222.2750	222.4824	226.5663	223.9658	0.8254	1.0027e-16
TRD	$\hat{\lambda}=0.9417$ $\hat{\alpha}=0.3524$	284.5310	284.7308	288.8163	286.2158	0.8842	1.8742e-16

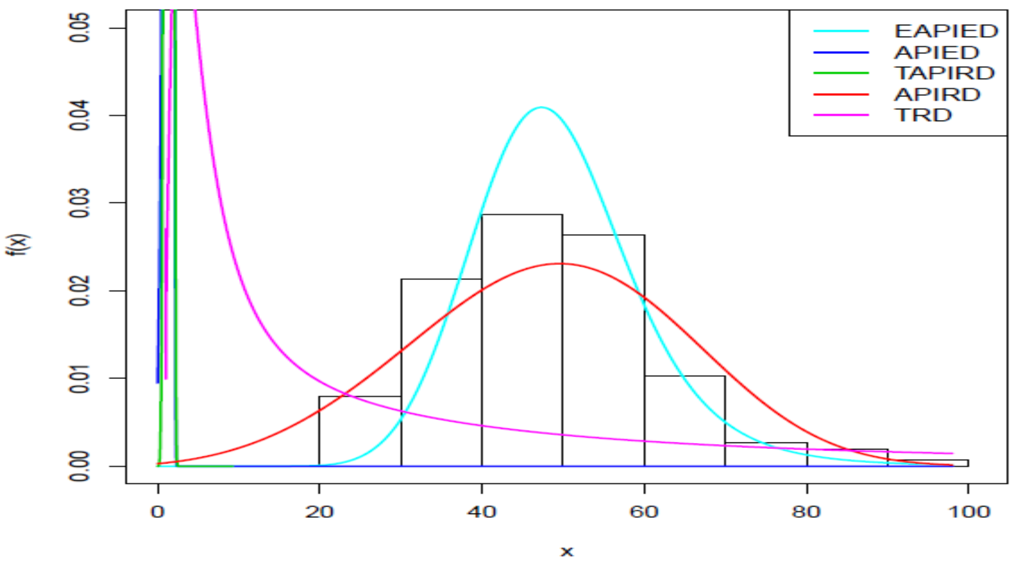

Figure 2: The empirical histogram of breast cancer

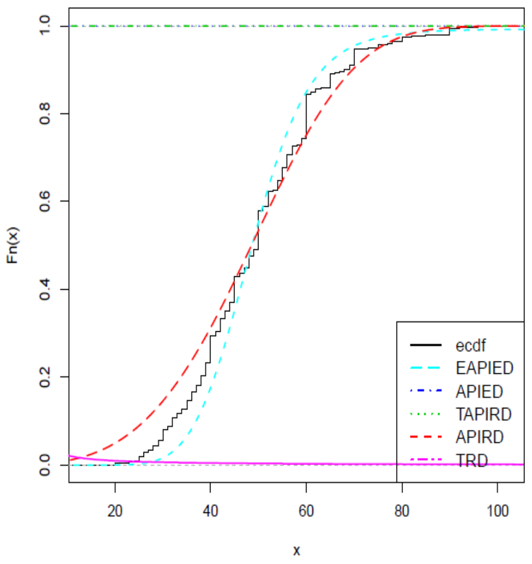

Figure 3: The empirical cdf of breast cancer

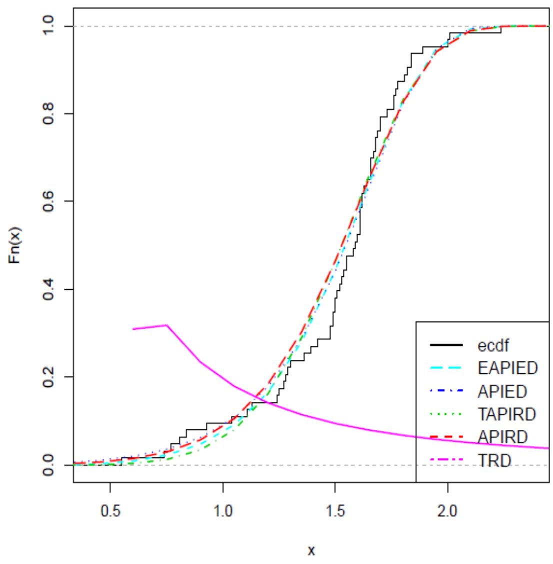

Figure 5: The empirical cdf of glass fibre

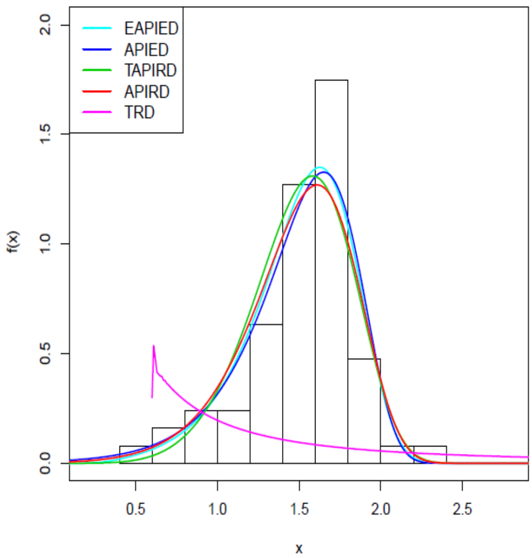

Figure 4: The empirical histogram of glass fibre

7. CONCLUSION

The E-APIE distribution has been proposed, derived and introduced. E-APIE distribution is presented as an improvement on APIE distribution for modelling real life time data. The distribution accommodates increasing and bathtub shaped hazard rate functions. The statistical properties were also established. The structure of the spread could be inverted bathtub or decreasing (depending on the value of the parameters). This density extends the APIED of the Unal *et al.* (2018). An application to a two real life data shows that the E-APIE distribution contends favourably with some existing families of distributions.

Conflicts of Interest

The Authors declare that there are no conflicts of interest.

Acknowledgement

The authors wish to acknowledge the Editor-in-Chief and the Anonymous reviewer(s) for their time and patient to improve this article through their meaningful contributions.

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