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Faster Fixed Point Iterative Scheme for Contraction Mappings

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ABSTRACT

In this paper, it is our aim to construct and study a new iteration scheme for the approximation of fixed points of several contraction mappings in the sense of Berinde. We prove strong convergence result for the suggested scheme under Lipchitz conditions. In addition, we demonstrate numerical simulations through tables and graphs displays to show that our suggested innovative scheme converges faster than many previously introduced iterative schemes for this mapping also in the sense of Berinde. Furthermore, a stability analysis under perturbation test is carried out. The results shows that with any given perturbation, the perturbed sequences also converges to the fixed of the mapping under investigation same as the unperturbed sequence. Our findings shows the effectiveness of our suggested scheme and can be used in approximating fixed points of contraction mappings and its applications in the field of epidemiology, engineering and computer science.

1. INTRODUCTION

Solution to nonlinear problems of the form $\Omega(\omega) = 0$ which appear in many applications like field of Engineering, Optimization theory, Computer Science and Differential Equations, mathematical modelling of epidemiology, etc has been reduced to the problem of fixed point of this form $\Omega(\omega) = \omega$, where is a given suitable operator and ω is the point that fixes the operator under transformation and without being changed or altered. This point if obtained after several transformation through sequence of iterations becomes the corresponding solution of problesms of this form $\Omega(\omega) = 0$. Many well known iteration techniques have been used to provide solution to this problem (see for exmple Picard, 1890, Mann, 1953, Ishikawa, 1974, Krasnosel'skii, 1955) and other classical approaches and thorems on fixed point (Browder, 1965, Zamfirescu, 1972).

Numerical computations have long shown that this methods are implementable but have a very slow convergence rate. This is a limiting problem when it comes to practical deployment in solving real-life problems. To overcome this limitation, many authors in recent time have resorted to formulating hybrid methods as a better iterative schemes. According to Okeke *et al.* (2025), modified iterative schemes are developed to solve nonlinear equations, integral equations, and optimization problems more efficiently. These modifications often introduce control parameters (e.g., step sizes) or combine

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multiple iterates to improve convergence rates.

Despite the successful deployment and adoption of hybrid iterative schemes to achieve convergence results, it has been observed that many of these schemes have shown different convergence decay and attributes subject to the class of problem and or the contraction condition (Abbas and Nazir, 2014, Berinde, 2002, 2004, 2007, Eke and Akewe, 2019, Ekuma-Okereke *et al.*, 2024, Jia, 2022, Okeke and Ofem, 2022, Okeke and Abbas, 2017, Phuengrattana and Suantai, 2010, Rhoades and Xue, 2010, Yu, *et al.*, 2021). Khan (2013) introduced an iteration process, called the Picard-Mann hybrid iterative process, which is a hybrid of Picard and Mann iterative schemes and observed that it converges faster than all the classical methods mentioned in the class of Berinde for contractions. Agarwal *et al.* (2007), introduced and studied the S-iterative scheme and their scheme was shown to converge faster than the Picard iterative scheme for contraction mappings.

Okeke *et al.* (2024) proposed and studied a four-step iterative scheme called Picard-Noor hybrid scheme which is a combination of Picard and Noor schemes. They showed that this scheme converged to the unique fixed point of a contraction mapping. Additionally, they provided a numerical illustration to demonstrate a faster convergence of their scheme and applied it in solving delay differential equations. In what follows, Ekuma-Okereke *et al.* (2024) introduced a hybrid iterative scheme of four-steps involving Picard and Mann combinations which converged to the unique fixed point of a contarcton mapping in the sense of Berinde. Their scheme performed better than other existing schemes including that of Okeke *et al.* (2024). Very recently, Okeke *et al.* (2025) introduced a fast iterative scheme and establish its convergence under a contractive condition. Their new scheme was shown numerically to be recent generalization of other previous done work with an application to simulate Ebola disease dynamics.

Notwithstanding these research achievement, there is still search for efficient iterative schemes with better rates of convergence and would have a practical application in solving real-life problems. This, therefore, is the driving factor towards this research. Hence, the research question: *Can we construct an iterative scheme which not only will converge to the unique point of any contraction mapping but will also show a better rate of convergence when compared with other recently announced schemes in this direction?*

It is our aim in this study to tackle this question affirmatively through a new iterative design under contraction condition.

2. MATERIALS AND METHODS

Let **K** be a non-empty and convex subset of a uniformly convex Banach space denoted by $\boldsymbol{X}$. Let $\boldsymbol{\Omega}: \mathbf{K} \to \mathbf{K}$ be a self contraction mapping defined on **K** and having values on **K.** Let the set of fixed points of this mapping be represented by $\boldsymbol{Fix}(\boldsymbol{\Omega}) := \{\boldsymbol{\omega} \in \boldsymbol{K}: \boldsymbol{\Omega}(\boldsymbol{\omega}) = \boldsymbol{\omega}\}$. Let $\boldsymbol{\omega_0} \in \boldsymbol{K}$ be the initial quess. Let $\boldsymbol{\delta_n}, \boldsymbol{\rho_n} \in [\mathbf{0}, \mathbf{1}]$ be two control parameters which can be a constant or a sequence of real numbers. The suggested scheme generates a sequence $\{\boldsymbol{\omega}\}_{\boldsymbol{n=0}}^{\infty} \in \boldsymbol{Fix}(\boldsymbol{\Omega})$ is expected to converge to the fixed point set also. In the process, each step makes use of the previous iterates to achieve a next and better step until converge is achieved after satisfying a given stopping criterion. In addition, we prove its convergence, show its stability and data dependance and provide numerical illustration to compare with other existing methods.

Suggested Method:

$$Require: \delta_n, \rho_n \in [0,1] \; and \; \omega_0 \in K.$$
$$Set \; \varepsilon > 0 \; arbitrarily \; small$$
$$for \; n = 0,1,2,\dots, Max, do$$
$$a_n := (1-\delta_n)\omega_n + \delta_n \Omega(\omega_n)$$
$$\vec{a}_n := \Omega(a_n)$$
$$b_n^{\cdot} := \Omega(\Omega(\vec{a}_n))$$
$$\vec{b}_n := (1-\rho_n)b_n + \rho_n \Omega(b_n)$$
$$c_n := \Omega(\vec{b}_n)$$
$$\omega_{n+1} := \Omega(\Omega(c_n))$$
$$If \; ||\omega_{n+1} - \omega_n|| < \varepsilon \; then$$
$$break$$
$$end \; if$$
$$end \; for.$$

3. RESULTS

Theorem 3.1: Let K be a nonempty closed uniformly convex subset of a Banach space $\boldsymbol{X}$ and let $\boldsymbol{\Omega}: \mathbf{K} \to \mathbf{K}$ be a self contraction mapping defined on $\mathbf{K}$ satisfying the contractive condition and such that $Fix(\Omega) := \{\omega \in K : \Omega(\omega) = \omega\} \neq \emptyset$. Let $\{\omega\}_{n=0}^{\infty} \in Fix(\Omega)$ be a sequence generated by the hybrid iterative process defined as follows:

$$Require: \delta_n, \rho_n \in [0,1] \; and \; \omega_0 \in K.$$
$$Set \; \varepsilon > 0 \; arbitrarily \; small$$
$$for \; n = 0,1,2,\dots, Max, do$$
$$a_n := (1-\delta_n)\omega_n + \delta_n \Omega(\omega_n)$$
$$\vec{a}_n := \Omega(a_n)$$
$$b_n^{\cdot} := \Omega(\Omega(\vec{a}_n))$$
$$\vec{b}_n := (1-\rho_n)b_n + \rho_n \Omega(b_n)$$
$$c_n := \Omega(\vec{b}_n)$$
$$\omega_{n+1} := \Omega(\Omega(c_n))$$
$$If \; ||\omega_{n+1} - \omega_n|| < \varepsilon \; then$$
$$break$$
$$end \; if$$
$$end \; for,$$

Then the sequence $\{\omega\}_{n=0}^{\infty}$ converges to a unique fixed point of $Fix(\Omega) := \{\omega \in K : \Omega(\omega) = \omega\}$.

Proof: Assume that $\sum_{n=0}^{\infty} \delta_n = \infty$, $p \in Fix(\Omega)$. Recall that a mapping is called a contraction mapping if there exists a constant $0 \leq c < 1$ such that

$$||\Omega(\mu) - \Omega(\vartheta)|| \leq ||\mu - \vartheta||, \forall \mu, \vartheta \in K.$$

Then following from the same approach of Ekum-Okereke *et al.* (2024), the conclusion of Theorem 3.1 holds. ■

Theorem 3.2: Let K be a nonempty closed uniformly convex subset of a Banach space $\boldsymbol{X}$ and let $\boldsymbol{\Omega}: \mathbf{K} \to \mathbf{K}$ be a self contraction mapping defined on $\mathbf{K}$ satisfying the contractive condition. Let $\{\omega\}_{n=0}^{\infty} \in Fix(\Omega)$ be a sequence generated by the above scheme which converges to $p \in Fix(\Omega)$. Then the suggested scheme is $\boldsymbol{\Omega}$-stable

Proof:

4. NUMERICAL COMPUTATIONS

Example 4.1: Let $\boldsymbol{K}$ be a subset of $\boldsymbol{R}$ and consider the following affine mapping $\boldsymbol{\Omega}: \boldsymbol{K} \to \boldsymbol{K}$ defined by

$$\Omega(\omega) = \frac{3}{4}\omega + 1, \forall \omega \in R.$$

It was verified in Okeke *et al.* (2025) that $\Omega(\omega) = \frac{3}{4}\omega + 1$ is a contraction mapping with contraction constant $c = \frac{3}{4}$ and the fixed point set $Fix(\Omega) := \{4\}$. Now, using arbitary initial point 4.5 $\in \boldsymbol{K}$, we have the comaparison results shown in the Table 4.1 and Fig. 4. 1 respectively.

n	NewMethod[n]	Okeke[n]	HybridPicMann[n]	PicNoor[n]
0	4.5000000	4.5000000	4.5000000	4.5000000
1	4.0587468	4.1131420	4.2011414	4.2428894
2	4.0069024	4.0256022	4.0809157	4.1179905
3	4.0008110	4.0057934	4.0325510	4.0573173
4	4.0000953	4.0013109	4.0130947	4.0278435
5	4.0000112	4.0002966	4.0052678	4.0135258
6	4.0000013	4.0000671	4.0021191	4.0065705
7	4.0000002	4.0000152	4.0008525	4.0031918
8	4.0000000	4.0000034	4.0003429	4.0015505
9	4.0000000	4.0000008	4.0001380	4.0007532

Table. 4.1: Comparison with other schemes for Example 4.1

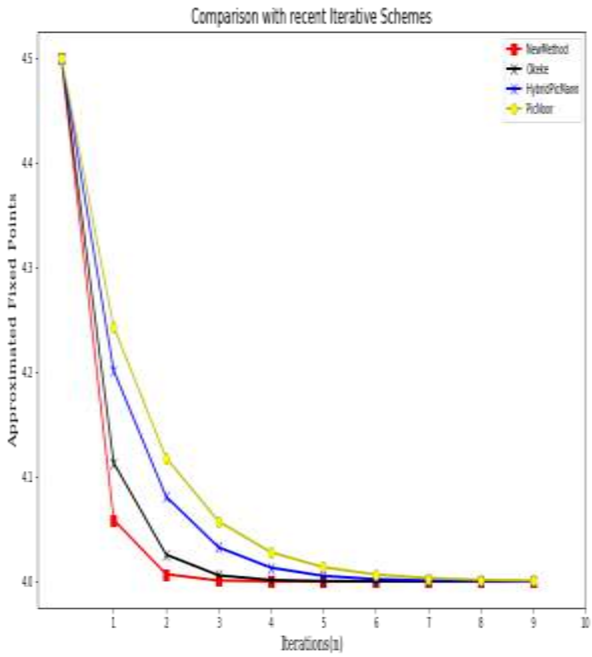

Fig. 4.1: Comparison with other schemes for Example 4.1

Observation 1: Table 4.1 and Fig. 4.1 reveals a visible comparison of four iterative schemes. It shows a faster convergence of our suggested new scheme after 9 iterations with a stopping criterion $||\omega_{n+1} - \omega_n|| < 10^{-7}$, which authenticates its superiority.

Table 4.2: Convergence behavious via residual norms for Example 4.1

n	NewMethod_Res[n]	Okeke_Res[n]
0	0.1250000	0.1250000
1	0.0146867	0.0282855
2	0.0017256	0.0064006
3	0.0002027	0.0014483
4	0.0000238	0.0003277
5	0.0000028	0.0000742
6	0.0000003	0.0000168
7	0.0000000	0.0000038
8	0.0000000	0.0000009
9	0.0000000	0.0000002

n	HybridPicM_Res[n]	PicNoor_Res[n]
0	0.1250000	0.1250000
1	0.0502853	0.0607224
2	0.0202289	0.0294976
3	0.0081377	0.0143293
4	0.0032737	0.0069609
5	0.0013169	0.0033814
6	0.0005298	0.0016426
7	0.0002131	0.0007980
8	0.0000857	0.0003876
9	0.0000345	0.0001883

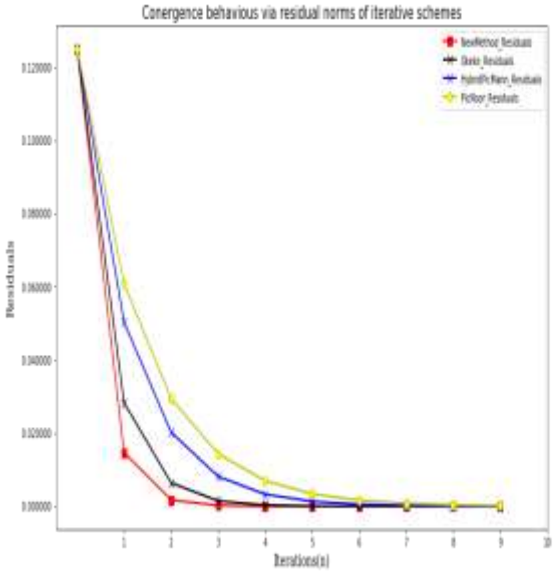

Fig 4.2: Convergence behavious via residual norms for Example 4.1

Observation 2: Table 4.2 and Fig. 4.2 reveals a visible comparison of four iterative schemes via residual norm $||\omega_{n+1} - \omega_n|| < 10^{-7}$. It shows a faster convergence of our suggested new scheme after 9 iterations with a stopping criterion$||\omega_{n+1} - \omega_n|| < 10^{-7}$, which authenticates its superiority.

Observation 3: Table 4.3 and Fig. 4.3 reveals a visible comparison of our iterative scheme via stability analysis under perturbations. We provide a slight perturbation of the original point of four variations by $\pm 0.1, \pm 0.2$ to 4.5. It shows that with any given perturbation, the perturbed sequences also converges to the fixed of the mapping same as the unperturbed sequence. Hence this suggests that our scheme is stable.

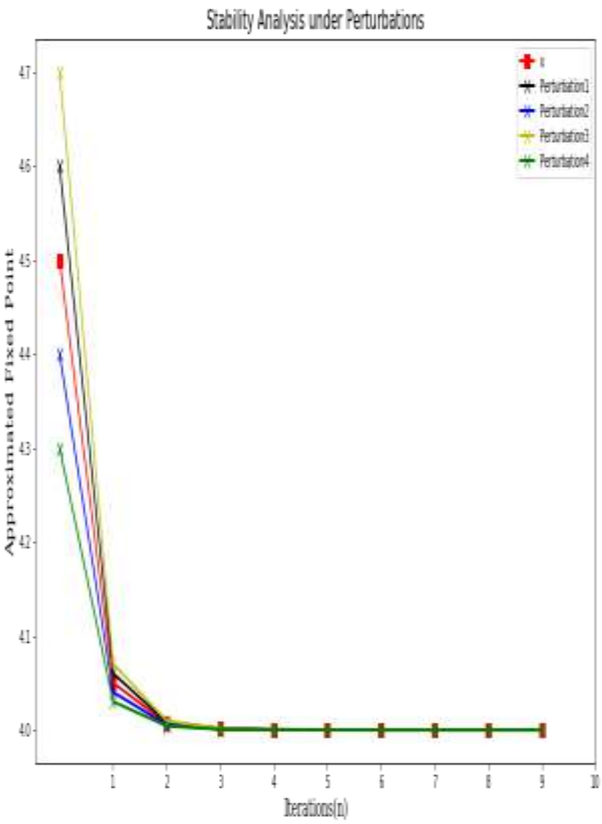

Fig 4.3: Stability analysis of new method under perturbations via Example 4.1

Table 4.3: Stability analysis of new method via Example 4.1

n	x[n]	Pert_1[n]	Pert_2[n]
0	4.5000000	4.6000000	4.4000000
1	4.0500565	4.0600677	4.0400452
2	4.0068209	4.0081851	4.0054567
3	4.0010201	4.0012241	4.0008161
4	4.0001596	4.0001915	4.0001277
5	4.0000256	4.0000308	4.0000205
6	4.0000042	4.0000050	4.0000034
7	4.0000007	4.0000008	4.0000006
8	4.0000001	4.0000001	4.0000001
9	4.0000000	4.0000000	4.0000000

n	Pert_3[n]	Pert_4[n]
0	4.7000000	4.3000000
1	4.0700790	4.0300339
2	4.0095493	4.0040926
3	4.0014281	4.0006120
4	4.0002234	4.0000957
5	4.0000359	4.0000154
6	4.0000059	4.0000025
7	4.0000010	4.0000004
8	4.0000002	4.0000001
9	4.0000000	4.0000000

Initial Guess
x[n]: original x(0) : 4.5000000
Perturbation (Pert_1) : +0.1000000
Perturbation (Pert_2) : - 0.2000000
Perturbation (Pert_3) : +0.2000000
Perturbation (Pert_4) : - 0.2000000

Example 4.2: Let $\boldsymbol{K} = [\mathbf{0}, \mathbf{1}]$ be a subset of $\boldsymbol{R}$ and consider the following nonlinear mapping $\boldsymbol{\Omega}: \boldsymbol{K} \to \boldsymbol{K}$ defined by

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \mathbf{cos}(\boldsymbol{w}) \quad \forall \boldsymbol{\omega} \in \boldsymbol{K}.$$

It was verified in Okeke *et al.* (2025) using the mean-value theorem that $\boldsymbol{\Omega}(\boldsymbol{\omega}) = \mathbf{cos}(\boldsymbol{w})$ is a contraction mapping with contraction constant $\boldsymbol{c} = \mathbf{sin}\ (\mathbf{1})$ and the fixed point set $\boldsymbol{Fix}(\boldsymbol{\Omega}) \coloneqq \{\mathbf{0.7390851332}\}$. Now, using arbitary initial point $0.9 \in \boldsymbol{K}$, we have the comaparison results shown in the Table 4.4 and Fig. 4. 4 respectively.

Table. 4.4: Comparison with other schemes for Example 4.2

n	NewMethod[n]	Okeke[n]
0	0.9000000000000	0.9000000000000
1	0.7431714830750	0.7315495784890
2	0.7391891047090	0.7394111496807
3	0.7390877784478	0.7390709694551
4	0.7390852005148	0.7390857484480
5	0.7390851349274	0.7390851064910
6	0.7390851332587	0.7390851343760
7	0.7390851332163	0.7390851331647
8	0.7390851332152	0.7390851332174
9	0.7390851332152	0.7390851332151

n	HybridPicMann[n]	PicNoor[n]
0	0.9000000000000	0.9000000000000
1	0.8129419541087	0.6718768667606
2	0.7356705905478	0.7655864650465
3	0.7392330728438	0.7283691941010
4	0.7390787063805	0.7433762730049
5	0.7390854123790	0.7373599521543
6	0.7390851210890	0.7397776180297
7	0.7390851337419	0.7388069940501
8	0.7390851331923	0.7391968203579
9	0.7390851332162	0.7390402804988

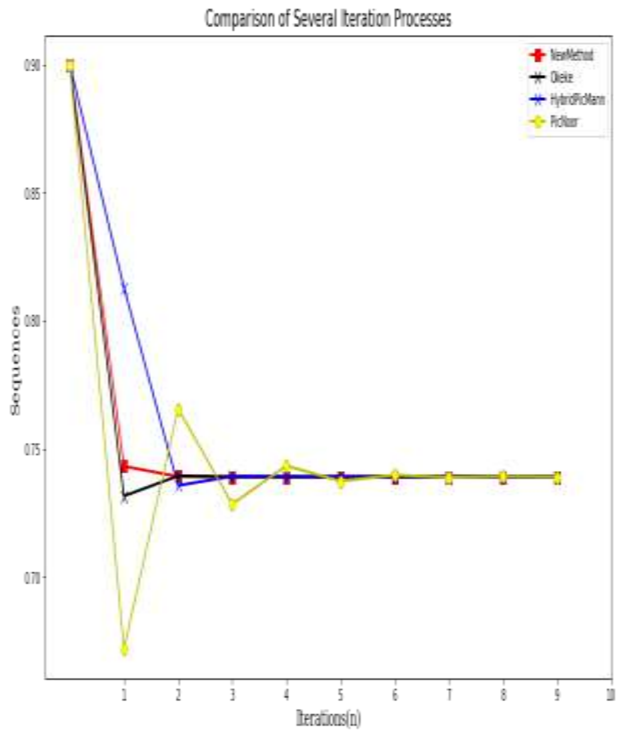

Fig. 4.2: Comparison with other schemes for Example 4.2

Observation 4: Table 4.4 and Fig. 4.4 reveals a visible comparison of four iterative schemes. Even for a nonlinear mapping, our scheme shows a faster convergence to the fixed point after 9 iterations with a stopping criterion $||\omega_{n+1} - \omega_n|| < 10^{-13}$, which authenticates its superiority also.

Example 4.3: Let $\boldsymbol{K} = [\mathbf{1}, \mathbf{10}]$ be a subset of $\boldsymbol{R}$ and consider the following cube root of affine mapping $\boldsymbol{\Omega}: \boldsymbol{K} \to \boldsymbol{K}$ defined by

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \sqrt[3]{\mathbf{4}\boldsymbol{\omega} + \mathbf{15}}\ \forall \boldsymbol{\omega} \in \boldsymbol{K}.$$

It was proved in Ekuma-Okereke *et al.*(2024) that $\boldsymbol{\Omega}(\boldsymbol{\omega}) = \sqrt[3]{\mathbf{4}\boldsymbol{\omega} + \mathbf{15}}$ is a contraction mapping with contraction constant $\boldsymbol{c} = \frac{\mathbf{1}}{\sqrt[3]{\mathbf{15}}} \in [\mathbf{1}, \mathbf{10}]$ and the fixed point set $\boldsymbol{Fix}(\boldsymbol{\Omega}) := \{\mathbf{3.0}\}$. Now, using arbitary initial point $10 \in \boldsymbol{K}$, we have the comaparison results shown in the Table 4.5 and Fig. 4. 5 respectively.

Table. 4.5: Comparison with other schemes for Example 4.3

n	NewMethod[n]	Okeke[n]
0	10.0000000000000	10.000000000000
1	3.0000196343862	3.0032381142531
2	3.0000000000684	3.0000019916240
3	3.0000000000000	3.0000000012252
4	3.0000000000000	3.0000000000008
5	3.0000000000000	3.0000000000000
6	3.0000000000000	3.0000000000000
7	3.0000000000000	3.0000000000000
8	3.0000000000000	3.0000000000000
9	3.0000000000000	3.0000000000000

n	HybridPicMann[n]	PicNoor[n]
0	10.0000000000000	10.0000000000000
1	3.2914804071136	3.6491066540685
2	3.0138692718231	3.0727639368638
3	3.0006645136624	3.0083384528766
4	3.0000318492296	3.0009579783003
5	3.0000015265143	3.0001100911481
6	3.0000000731650	3.0000126521308
7	3.0000000035068	3.0000014540409
8	3.0000000001681	3.0000001671051
9	3.0000000000081	3.0000000192045

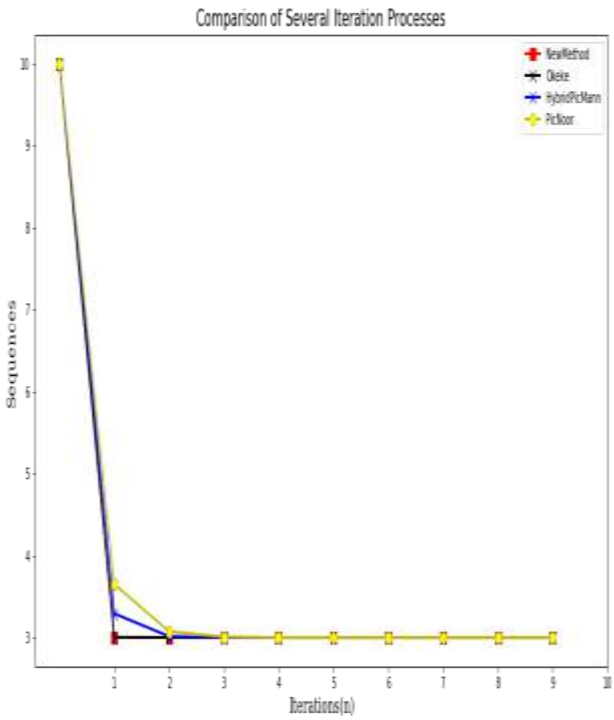

Fig. 4.5: Comparison with other schemes for Example 4.3

Observation 5: Table 4.5 and Fig. 4.5 reveals a visible comparison of four iterative schemes. Even for a cube root of affine mapping, our scheme shows a faster convergence to the fixed point after 4 iterations with a stopping criterion $||\omega_{n+1} - \omega_n|| < 10^{-13}$, which authenticates its superiority also.

5. CONCLUSION

In this paper, we constructed and suggested a faster iteration scheme for the approximation of fixed points of several contraction mappings in the sense of Berinde. Our results showed strong convergence and s faster convergence than many recently suggested iterative schemes for this mapping also. Furthermore, under perturbation test, our scheme is stable under transformation.

Conflicts of interest

The authors declared that there is no conflict of interest.

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