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On the Extended New Generalized Exponential Distribution: Properties and Applications

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Abstract
Lifetime processes has received several attentions recently through modeling the way and manner in which they are distributed. In this article, we propose an extended new generalized exponential distribution for a lifetime processes. The statistical properties of distribution such as kurtosis, survival, hazard, cumulative, odd functions, quantiles, skewness, reversed hazard, and order statistics are derived. The parameters of this class of distribution were also obtained by maximum likelihood method. The behaviour of the model was studied through simulation. Finally, a real life data was used to examine the performance of the propose model. The results show that the model perform favourably well with existing continuous models.

Keywords: Length biased distribution, Maximum likelihood Estimation, Odd functionsWeighted exponential distribution.

1. Introduction

There have been several attempts to develop new families of exponential distribution with greater flexibility in modeling life time data. These newly developed exponential distributions are developed either by adding a new parameter, compounding two or more distributions or building a new distinct one; but introducing a new parameter may be more flexible.

The exponential distribution has been used in modeling continuous memoryless random processes with constant failure rate. This includes waiting time problem and machine learning in big data environment. It is important to note that the occurrence of this constant failure rate is almost impossible in real life. Hence, to account for this great disadvantage, Keller et al. (1982) proposed inverted exponential distribution with inverted bathtub hazard rate. Abouammoh and Alshingiti (2009) proposed the generalized inverted exponential distribution. The parameters of the generalized inverted exponential distribution were estimated in Dey et al. (2017). Transmuted inverse exponential distribution was applied in medicine and engineering in Oguntunde and Adejumo (2014). Oguntunde et al. (2014a) examined the statistical properties of the exponentiated generalized inverted exponential distribution. Oguntunde et al. (2014b) proposed the Kumaraswamy inverse exponential distribution. Olapade (2014) proposed the extended generalized exponential distribution. Pinho et al. (2015) proposed the Harris extended exponential distribution. Fatima and Roohi (2015) proposed the extended Poisson exponential distribution. Anake et al. (2015) proposed fractional beta exponential distribution. Thiago et al. (2016) proposed the exponentiated generalized extended exponential distribution. Nadarajah and Okorie (2017) proposed the moments of the alpha power transformed generalized exponential distribution. Alizadeh et al. (2017) proposed the generalized odd generalized exponential family of distributions. Mahdavi and Jabari (2017) proposed the extended weighted exponential distribution. Hamedani et al. (2018) proposed the type I general exponential class of distribution. Unal et al. (2018) proposed the alpha power inverted exponential distribution. Eghwerido et al. (2019) proposed the Gompertz alpha power inverted exponential distribution. Hassan et al. (2019) proposed the odd generalized exponential power function distribution.

Let the random variable X represents the waiting time between successive events with a Poisson distribution of mean λ. Then, the random variable X follows an exponential with the probability distribution function (pdf) given as

$$f_X(x) = \lambda \exp(-\lambda x), \quad x > 0, \lambda > 0. \qquad (1)$$

The cumulative distribution function (cdf) of the corresponding exponential distribution is given as

$$F_X(x) = 1 - \exp(-\lambda x) \quad x > 0, \lambda > 0 \qquad (2)$$

More so, Gupta and Kundu (2001) proposed the generalized exponential (GE) distribution with the pdf given as

$$f_X(x) = \alpha\lambda(1 - \exp(-\lambda x))\exp(-\lambda x) \quad x > 0, \alpha > 0, \lambda > 0 \qquad (3)$$

with the corresponding cdf given as

$$F_X(x) = (1 - \exp(-\lambda x))^{\alpha} \; x > 0, \alpha > 0, \lambda > 0 \qquad (4)$$

where α and λ are the shape and scale parameter respectively.

In this study, the extended new generalized exponential distribution with its statistical properties will be studied intensively.

The article is organized as follows: The introduction was given in section 1. Section 2 is the formulation of the ENEGE distribution. In section 3, we discuss the statistical properties of the ENEGE distribution, section 4 is the parameter estimations of the proposed distribution. Section 5 is the simulation study of the proposed distribution. Section 6 is the application of the proposed distribution to real life data to examine its flexibility. Section 7 is the conclusion.

2. A Three Parameter Class of the Extended New Generalized Exponential Distribution

In this article, we shall propose a three parameter class of distribution called extended new generalized exponential distribution called ENEGE distribution.

Let X be a random variable of the continuous type, then the probability distribution function of the ENEGE distribution is given as

$$f_{ENEGE}(x) = \frac{\alpha\lambda}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)} \sum_{k=0}^{\alpha-1} \beta^{k} \binom{\alpha-1}{k} \times \exp(-\lambda x(\alpha-k)) \qquad x > 0,\ \alpha > 1,\ \beta > 1,\ \lambda > 0 \tag{5}$$

provided $\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) \neq 0$ and α is the shape parameter, β is the extension parameter and λ is the scale parameter.

Thus, the ENEGE distribution is expressed as a sum of exponential distribution with parameters $\lambda(\alpha-k)$.

Figures 1 and 2 show the plots of pdf and cdf for the ENEGE distribution for various values of parameters

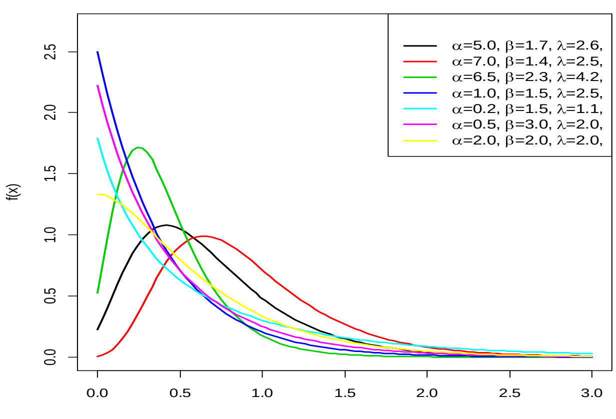

Figure 1: The pdf of the ENEGE distribution for different parameter values

Remark 1

From Figure 1, the shape of the ENEGE distribution could be increasing or decreasing depending on the value of the parameters. Also, it could be left skewed.

The cdf of the ENEGE distribution is given as

$$F_{ENEGE}(x) = \frac{1}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)} \sum_{k=0}^{\alpha} (-1)^{\alpha} \binom{\alpha}{k} \times \beta^{\alpha-k} \left(\exp(-\lambda\alpha x) - 1\right);\ x > 0,\ \alpha > 1,\ \beta > 1,\ \lambda > 0 \tag{6}$$

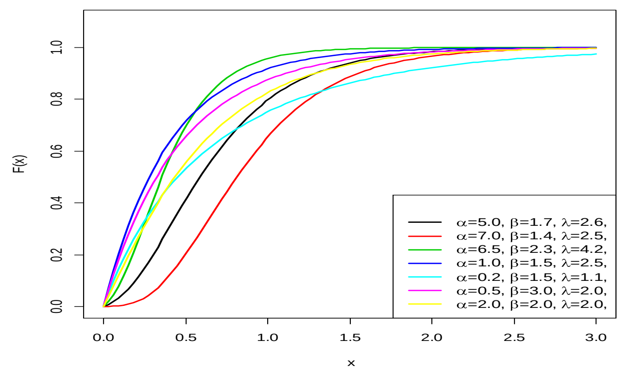

Figure 2: The cdf of the ENEGE distribution for different parameter values

Remark 2

Figure 2 is the shape of the cdf of the ENEGE distribution. It shows that it is increasing depending on the value of the parameters.

The following special models can be deduced from the pdf of the ENEGE distribution: for

- $\beta = 1$ we obtain the generalized exponential distribution,
- $\alpha = 1$ we obtain the exponential distribution with parameter λ

2.1 Reliability Analysis

The reliability function $R(x)$ of the ENEGE distribution is given as

$$R(x)=\frac{\beta^{\alpha}-\sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha}\exp(-\lambda\alpha x)}{\left(\beta^{\alpha}-(\beta-1)^{\alpha}\right)};\quad x>0,\ \alpha>1,\ \beta>1,\ \lambda>0 \tag{7}$$

Figure 3 shows the reliability of the ENEGE distribution.

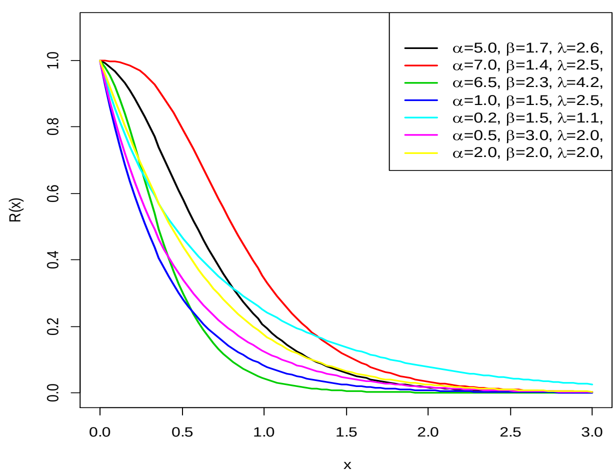

Figure 3: The reliability function of the ENEGE distribution for different parameter values.

Remark 3
Figure 3 is the shape of the reliability function of the ENEGE distribution. It shows that it is bathtub and decreasing depending on the value of the parameters.

2.2 Hazard Rate Function

The hazard rate function $h(x)$ of the ENEGE distribution is given as

$$h(x)=\frac{\alpha\lambda\sum_{k=0}^{\alpha-1}(-1)^{\alpha-1}\binom{\alpha-1}{k}\beta^{\alpha-1}\exp(-\lambda x(\alpha-k))}{\beta^{\alpha}-\sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha}\exp(-\lambda\alpha x)};\quad x>0,\ \alpha>1,\ \beta>1,\ \lambda>0 \tag{8}$$

Figure 4 shows the hazard rate function of the ENEGE distribution.

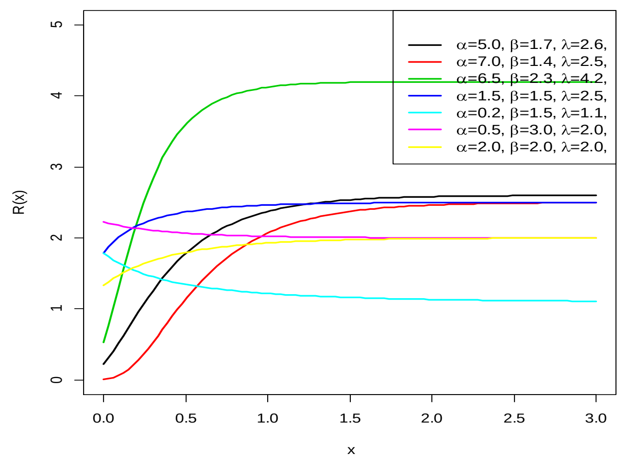

Figure 4: The hazard rate function of the ENEGE distribution for different parameter values.

Remark 3
It can be deduced from Figure 4 that the shape of the hazard function of the ENEGE distribution could be constant, bathtub, increasing, or decreasing (depending on the value of the parameters).

2.3 Cumulative Hazard Rate

The cumulative hazard rate function $H(x)$ of the ENEGE distribution is given as

$$H(x) = -In\left(\frac{\beta^{\alpha} - \sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha}\exp(-\lambda\alpha x)}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)}\right)$$

$$= In\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) - In\left(\beta^{\alpha} - \sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha}\exp(-\lambda\alpha x)\right)$$

2.4 Reversed Hazard Function

The reversed hazard function $r(x)$ of the ENEGE distribution is given as

$$r(x) = \frac{\lambda\alpha\sum_{k=0}^{\alpha-1}\beta^{k}\binom{\alpha-1}{k}\exp(-\lambda x(\alpha-k))}{\sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha-k}(\exp(-\lambda\alpha x)-1)}$$

2.5 Odds Function

The Odds function $O(x)$ of the ENEGE distribution is given as

$$O(x) = \frac{\sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha-k}(\exp(-\lambda\alpha x)-1)}{\beta^{\alpha} - \sum_{k=0}^{\alpha}(-1)^{\alpha}\binom{\alpha}{k}\beta^{\alpha}\exp(-\lambda\alpha x)}$$

3. Some mathematical properties of the extended new generalized exponential distribution formulation

In this section, we investigate some mathematical properties of the ENEGE distribution. This includes the quantile and random number generation, probability weighted moments, entropies, moments of the residual and reversed lifes and order statistics.

a. Quantile function and random number generation

Let X be a random variable such that $X \sim ENEGE(\alpha, \beta, \lambda)$. Then, the quantile function of X for $u \in [0,1]$ is obtained as

$$x_p = -\lambda^{-1} \times \log\left(\beta + \left[u\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) + (\beta-1)^{\alpha}\right]^{\frac{1}{\alpha}}\right) \quad 0 < u < 1 \tag{9}$$

Now, for $u = 0.5$, we have the median M of the random variable X. Thus, the random number of the ENEGE density function can be generated using Equation (9). However, the 25^{th} percentile and the 75^{th}percentile are given respectively as

$$Q_1 = -\lambda^{-1} \times \log\left(\beta + \left[0.25\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) + (\beta-1)^{\alpha}\right]^{\frac{1}{\alpha}}\right); \quad 0 < u < 1 \tag{10}$$

$$Q_3 = -\lambda^{-1} \times \log\left(\beta + \left[0.75\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) + (\beta-1)^{\alpha}\right]^{\frac{1}{\alpha}}\right); \quad 0 < u < 1 \tag{11}$$

The Moors' kurtosis for the ENEGE distribution is given as

$$K_s = \frac{Q\left(\frac{6}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{2}{8}\right)} \tag{12}$$

The Bowley's skewness for the ENEGE distribution is given as

$$S_k = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \quad (13)$$

i. Moments

The r^{th} moment of the random variable X is given as

$$\mu_r^{'} = \sum_{k=0}^{\alpha-1} \frac{\alpha\lambda}{\beta^{\alpha} - (\beta-1)^{\beta}} \times \int_0^{\infty} \binom{\alpha-1}{k} x^r \exp(-\lambda x(\alpha-k))dx$$
$$= \sum_{k=0}^{\alpha-1} \frac{\alpha\lambda\Gamma(r+1)}{\left(\beta^{\alpha} - (\beta-1)^{\beta}\right)(\lambda(\alpha-k))} \binom{\alpha-1}{k} \quad (14)$$

The n^{th} central moment of the random variable X, say μ_n is given as

$$\mu_1 = E(X-\mu)^n = \sum_{p=0}^{n} (-1)^p \binom{n}{p} \mu_1^{'p} \mu_{n-p}^{'} \quad (15)$$

ii. Distribution of the order statistics

Let $x_1, x_2, \cdots, x_n$ be a random sample from an infinite population with the ENEGE distribution at the point x then the pdf of the k^{th} order statistics is given as

$$f_{i:n}(x) = \frac{n!}{(k-1)!(n-k)!} \left(F_{ENEGE}(x)\right)^{k-1} \times f_{ENEGE}(x)\left(1 - F_{ENEGE}(x)\right)^{n-k} \quad -\infty < x < \infty \quad (16)$$

where $\Gamma(.)$ is a gamma function.

$$f_{i:n}(x) = \frac{n!}{(k-1)!(n-k)!} \times \left(\frac{1}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)} \sum_{k=0}^{\alpha} (-1)^{\alpha} \binom{\alpha}{k} \beta^{\alpha-k} \left(\exp(-\lambda\alpha x) - 1\right) \right)^{k-1}$$
$$\times \frac{\alpha\lambda}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)} \sum_{k=0}^{\alpha-1} \beta^{k} \binom{\alpha-1}{k} \exp(-\lambda x(\alpha-k))$$
$$\times \left(1 - \frac{1}{\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right)} \sum_{k=0}^{\alpha} (-1)^{\alpha} \binom{\alpha}{k} \beta^{\alpha-k} \left(\exp(-\lambda\alpha x) - 1\right) \right)^{n-k} \quad -\infty < x < \infty \quad (17)$$

b. Parameter Estimation for the extended new generalized exponential distribution formulation

Let $x_1, x_2, \cdots, x_n$ be a random sample from ENEGE distribution then, the vector of parameters of can be obtained as from the partial derivative of the likelihood function ℓ of the ENEGE distribution as

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} - n\left(\beta^{\alpha} In\beta - (\beta-1)^{\alpha} In(\beta-1)\right) + \sum_{i=1}^{n} \log\left(\beta - \exp(-\lambda x_i)\right) = 0 \quad (18)$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{n}{\alpha} + (\alpha-1) \sum_{i=1}^{n} \frac{x_i \exp(-\lambda x_i)}{\beta - \exp(-\lambda x_i)} - \sum_{i=1}^{n} x_i = 0 \quad (19)$$

$$\frac{\partial \ell}{\partial \beta} = \frac{n\left(\alpha\beta^{\alpha-1} - \alpha(\beta-1)^{\alpha-1}\right)}{\beta^{\alpha} - (\beta-1)^{\alpha}} + (\alpha-1)\sum_{i=1}^{n}\frac{1}{\beta - \exp(-\lambda x_i)} = 0 \quad (20)$$

The estimates of the unknown parameters are obtained through the Newton-Raphson algorithm in software like MATLAB, R, MAPLE, and so on. Thus, Setting the partial derivative to zero and solving the nonlinear equations yield the MLEs.

c. Simulation study

A simulation is carried out to test the flexibility and efficiency of the ENEGE distribution. The simulation is performed as follows:

- Data are generated using $x = -\lambda^{-1} \times \log\left(\beta + \left[u\left(\beta^{\alpha} - (\beta-1)^{\alpha}\right) + (\beta-1)^{\alpha}\right]^{\frac{1}{\alpha}}\right)$; $0 < u < 1$
- The values of the parameters are set as $\alpha = 0.2,5,2.0,2.5,5.0;$ $\beta = 1.2,1.5,3.0,8.0$ and $\lambda = 0.5,1.0,1.5.$

We evaluated the skewness (S_K), kurtosis (k_s), median (M), 25th percent (P) and 75th percent (P_K) of the ENEGE distribution. Table 1 shows the simulation for different values of parameters for the ENEGE distribution.

The values of λ, median and first quantile increase with constant skewness and kurtosis. Increase in β decreases the skewness, kurtosis, first and third quantiles. Increase in α beyond 1.5 makes skewness, kurtosis, median, first and third quantile negative.

i. Real life application

A gas fibers and carbon data real life datasets are applied to the proposed model to examine the performance of the model based on its statistic. Several criteria were used to determine the distribution for the best fit: Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Bayesian Information Criteria (BIC), and Hannan and Quinn Information Criteria (HQIC) were provided. The statistics are as follows:

$$AIC = 2\left(p - \hat{\ell}\right),$$

$$CAIC = \frac{2pn}{(n-p-1)} - 2\hat{\ell},$$

$$BIC = p\log(n) - 2\hat{\ell},$$

$$HQIC = 2p\log(\log(n)) - 2\hat{\ell},$$

where ℓ is the log-likelihood function, p is the number of model parameters and n is the sample size.

3.4 *Glass fibers data*

The first data were on 1.5 cm strengths of glass fibres obtained by workers at the UK National Physical Laboratory. The data has been used by Smith and Naylor(1987), Haq et al. (2016), Bourguinon et al. (2014), Merovci et al. (2016) , Rastogi and Oguntunde (2018), and Oguntunde et al. (2017). The observations are given as follows:

0.55, 0.74, 0.77, 0.81, 0.84, 1.24, 0.93, 1.04, 1.11, 1.13, 1.30, 1.25, 1.27, 1.28, 1.29, 1.48, 1.36, 1.39, 1.42, 1.48, 1.51, 1.49, 1.49, 1.50, 1.50, 1.55, 1.52, 1.53, 1.54, 1.55, 1.61, 1.58, 1.59, 1.60, 1.61, 1.63, 1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68, 1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82, 1.84, 1.84, 2.00, 2.01, 2.24.

Table 2 is the measure of comparison for the various distributions under consideration. We compare the ENEGE model with EGE model (Olapade 20014), Exponential model, Alpha power inverted exponential model (APIE), Unal et al., (2018), Frechet model (Fr), Transmuted Frechet model (TFr), gamma extended Frechet (GEFr), beta Frechet (BFr), Afify et al. (2016).

Table 1: Simulation results of S_k , K_s , M , P, P_k for the ENEGE distribution for different parameter values

		$\lambda=0.5$					$\lambda=1.0$					$\lambda=1.5$				
$\hat{\alpha}$	$\hat{\beta}$	S_k	K_s	M	P	P_k	S_k	K_s	M	P	P_k	S_k	K_s	M	P	P_k
0.2	1.2	0.1087	0.1817	-1.1	-0.9	-1.3	0.1087	0.1817	-0.5	-0.4	-0.7	0.1087	0.1817	-0.4	-0.3	-0.5
	1.5	0.0583	0.0958	-1.7	-1.6	-2.0	0.0583	0.0958	-0.9	-0.8	-1.0	0.0583	0.0958	-0.5	-0.5	-0.7
	3.0	0.0177	0.0286	-3.4	-3.3	-3.5	0.0177	0.0286	-1.7	-1.7	-1.7	0.0177	0.0286	-1.1	-1.1	-1.2
	5.0	0.0091	0.0147	-4.5	-4.4	-4.6	0.009	0.0147	-2.2	-2.2	-2.3	0.0091	0.0147	-1.5	-1.5	-1.5
	8.0	0.0053	0.0085	-13.7	-13.7	-13.7	0.0053	0.0085	-2.7	-2.7	-2.8	0.0053	0.0083	-1.8	-1.8	-1.8
1.5	1.2	-0.1341	-0.2147	-1.4	-1.1	-1.6	-0.1341	-0.2147	-0.7	-0.6	-0.8	-0.1341	-0.2147	-0.5	-0.5	-0.5
	1.5	-0.1049	-0.1655	-1.9	-1.7	-2.1	-0.1049	-0.1655	-0.9	-0.8	-1.0	-0.1049	-0.1655	-0.6	-0.6	-0.7
	3.0	-0.0472	-0.0743	-3.4	-3.3	-3.5	-0.0472	-0.0742	-1.7	-1.7	-1.8	-0.0472	-0.0743	-1.1	-1.1	-1.2
	5.0	-0.0269	-0.0426	-4.5	4.5	4.6	-0.0269	-0.0426	-2.3	-2.2	-2.2	-0.0269	-0.0426	-1.5	-1.5	-1.5
	8.0	-0.0164	-5.4828	-5.5	-5.4	-5.5	-0.0164	-0.0260	-2.7	-2.7	-2.8	-0.0164	-0.0260	-1.8	-1.8	-1.8
2.0	1.2	-0.1742	-0.2826	-1.4	-1.2	-1.6	-0.1742	-0.2826	-0.7	-0.6	-0.8	-0.1742	-0.2826	-0.5	-0.4	-0.5
	1.5	-0.1466	-0.2332	-1.9	-1.7	-2.1	-0.1466	-0.2332	-1.0	-0.9	-1.0	-0.1466	-0.2332	-0.6	-0.6	-0.7
	3.0	-0.0706	-0.1106	-3.4	-3.3	-3.5	-0.0706	-0.1106	-1.7	-1.7	-1.8	-0.0706	-0.1106	-1.1	-1.1	-1.2
	5.0	-0.0406	-0.0639	-4.5	-4.5	4.6	-0.0406	-0.0639	-2.3	-2.2	-2.3	-0.0406	-0.0639	-1.5	-1.5	-1.5
	8.0	-0.0246	-0.0390	-5.5	-5.5	-5.5	-0.0246	-0.0390	-2.7	-2.7	-2.8	-0.0246	-0.0390	-1.8	-1.8	-1.8
2.5	1.2	-0.1973	-0.3221	-1.5	-1.3	-1.6	-0.1973	-0.3221	-0.7	-0.6	-0.8	-0.1973	-0.3221	-0.5	-0.4	-0.5
	1.5	-0.1764	-0.2830	-2.0	-1.8	-2.1	-0.1764	-0.2830	-1.0	-0.9	-1.0	-0.1764	-0.2830	-0.7	-0.6	-0.7
	3.0	-0.0926	-0.1447	-3.4	-3.3	-3.5	-0.0926	-0.1447	-1.7	-1.7	-1.8	-0.0926	-0.1447	-1.1	-1.1	-1.2
	5.0	-0.0539	-0.0846	-4.5	-4.6	-4.6	-0.0539	-0.0846	-2.3	-2.2	-2.3	-0.0539	-0.0846	-1.5	-1.5	-1.5
	8.0	-0.0329	-0.0519	-5.5	-5.5	-5.5	-0.0329	-0.0519	-2.7	-2.7	-2.8	-0.0329	-0.0519	-1.8	-1.8	-1.8
5.0	1.2	-0.2341	-0.3829	-1.6	-1.5	-1.7	-0.2341	-0.3829	-0.8	-0.7	-0.8	-0.2341	-0.3829	-0.5	-0.5	-0.6
	1.5	-0.2320	-0.3788	-2.1	-2.0	-2.1	-0.2320	-0.3788	-1.0	-1.0	-1.1	-0.2320	-0.3788	-0.7	-0.6	-0.7
	3.0	-0.1762	-0.2779	-3.5	-3.4	-3.5	-0.1762	-0.2779	-1.7	-1.7	-1.8	-0.1762	-0.2779	-1.2	-1.1	-1.2
	5.0	-0.1147	-0.1787	-4.5	-4.5	-4.6	-0.1147	-0.1787	-2.3	-2.2	-2.3	-0.1147	-0.1787	-1.5	-1.5	-1.5
	8.0	-0.0725	-0.1133	-2.7	-2.7	-2.8	-0.0725	-0.1133	-2.7	-2.7	-2.8	-0.0725	-0.1133	-1.8	-1.8	-1.8

Table 2: Performance rating between distributions with glass fibres dataset

Distributions	AIC	CAIC	BIC	HQIC	-LL
ENEGE	67.6	143.9	151.8	146.6	30.8
BFr	68.6	69.3	77.2	77.0	30.3
GEFr	69.6	70.3	78.1	72.9	30.8
Fr	97.7	97.9	102	99.4	46.8
TFr	100.1	100.5	106.5	102.6	47.5
EGE	145.3	145.9	153.8	148.6	68.6
E	179.6	181.8	185.9	179.7	88.8

APIE	196.3	196.5	200.6	198.0	95.7

3.5 Carbon Data

Our second set of data is from Nichols and Padgett (2006). It consists of 100 observations taken on breaking stress of carbon fibres (in Gba). The dataset are as follow:

3.7, 2.74, 2.73, 2.5, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11,4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43,2.95, 2.97, 3.39, 2.96, 2.53,2.67, 2.93, 3.22, 3.39, 2.81, 4.2, 3.33, 2.55, 3.31, 3.31, 2.85,2.56, 3.56, 3.15, 2.35, 2.55, 2.59,2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19,1.57, 0.81, 5.56, 1.73, 1.59, 2, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69,1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.7, 2.03, 1.8, 1.57, 1.08, 2.03, 1.61, 2.12,1.89, 2.88, 2.82, 2.05, 3.65.

Table 3 is the goodness-of-fit and the performance rating of the ENEGE distribution using several test statistics for the carbon dataset.

Table 3: Performance rating of the ENEGE distribution with carbon dataset

Distributions	AIC	CAIC	BIC	HQIC	-LL
ENEGE	294.5	595.2	602.8	598.2	144.3
BFr	311.1	311.6	321.6	315.4	155.8
GEFr	312.0	312.4	332.4	316.2	156.2
Fr	348.3	348.4	353.5	350.4	174.2
TFr	350.5	350.7	358.3	353.6	175.4
EGE	420.0	418.3	425.8	421.2	206.0
E	394.7	394.7	397.3	395.8	196.4
APIE	417.8	418.0	423.1	419.9	206.9

3.6 Discussion

The performance of a model is determined by the value of the Akaike Information Criteria (AIC) with the lowest value is regarded as the best model. In the two real life cases considered, the ENEGE distribution has the lowest AIC value with 67.6 and 294.5 respectively. Also, the ENEGE distribution has the lowest value of log-likelihood of 30.8 and 144.3 respectively. Hence, it performs better than some existing models for the data used.

Conclusion

The ENEGE distribution has been derived. The basic statistical properties which include the order statistics distribution, cumulative hazard function, reversed hazard function, quantile, median, hazard function, odds function have been successfully established. The shape of the distribution could be increasing or decreasing (depending on the value of the parameters). An application to a two real life data shows that the ENEGE is a strong competitor compared to other continuous distributions.

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