

Time Varying Parameters and Curvature of Nigerian Eurobond

Gbenga M. Ogungbenle & Joshua S. Adeyele

Department of Actuarial Science, University of Jos, Nigeria
Email: gbengarising@gmail.com, adesolojosh@gmail.com

Abstract

The aim of this paper is to numerically estimate the time varying parameters and curvature of Nigerian Eurobond by employing the five months available data knowing on assumption that the estimated parameters are functionally dependent on the Nelson-Siegel model used for the estimation of the time varying parameters. The specific objectives of determine the level, slope and the curvature factor for the Nigerian Eurobond and derive an in-sample yield model at various maturities. It was discovered from the results obtained that the variabilities in the short term and long term differ significantly and the three principal components functionally associated by level, slope and curvature account for about 98.9% of the changes in term structure variability

Key words: Term structure, curvature, Eurobond, parameters

1. Introduction

Yield curve describing a graphical picture between yields and term to maturity presents a key determinant factor of bond market conditions. As a result, many functional techniques in actuarial-finance analysis have been deployed to model the yield curve (Christens et al., 2011; Antonio, et al., 2015). Hence the need to model the term structure of interest rate in Nigeria. Every financial computation in life insurance employs interest rate in actuarial valuation analysis of retirement benefits and annuity schemes. Interest rate further predicts the choice of life insurance portfolio where the insured's funds is invested so as to meet future contingencies. Insurance and pension fund assets and liabilities are subject to vagaries of interest rate because of

volatilities in interest rates risk exposure (Nelson & Siegel, 1987; Jens et al., 2017; Barry, 2017). Life insurance liabilities comprise of future promises to offset contingent claims on insured peril such as a disability, death in service, pensions and gratuity benefits whose effective maturity could be very long when compared to the underlying assets of the companies, hence, insurance companies can be exposed to risk from changes in longer-term interest rates. Interest rate which varies with changes in trends in financial market and responsible for interest rate risk, necessitates the need by fund managers to use a more accurate interest rate technique for computational purposes. Life insurance companies and pension fund administrators and custodians underwrite policies to cover future risks. Because the terms and conditions of these

contracts vary with respect to different time to maturity not included in the range of the Nigerian bonds, it may offer challenges to estimate the rates that may be used for contract of such time to maturity. A few advanced economies employ defined actuarial vehicles for modeling term structure to make it easier for their respective Central Banks to publish interest rate term structures on daily basis (Vasicek & Oldrich, 1977; Nymand-Andersen, 2018;) which unlike in Nigeria there is hardly any such technique.

2. Theory and Methodology

2.1 Nelson-Siegel Model

Nelson-Siegel (1987) and Svenson (1994) assume that the term structure of interest rate is a functional form consisting of four parameters. $\beta = \{\beta_{it}, \lambda\}, i = 0,1,2$. This model describes a convenient and parsimonious three-component decay estimations. Nelson and Siegel (1987) proposed the forward rate curve as:

$$f_t(\tau) = \beta_{0t} + \beta_{1t}e^{-\lambda_{1t}\tau} + \beta_{2t}\tau e^{-\lambda_{1t}\tau}$$

The Nelson–Siegel curve can be viewed to mean a constant parameter with a Laguerre function, representing the product of a non-monic polynomial and an exponential decay term. The associated yield curve obtained by integrating the forward rate function becomes,

$$y_t(\tau) = \beta_{0t} + \beta_{1t} \left[\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} \right] + \beta_{2t} \left[\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau} \right]$$

The Nelson–Siegel yield curve corresponds to a discount curve which starts at one with zero maturity and tends to zero at infinity maturity.

2.2 The Svenson Model (Extended Nelson-Siegel Model)

Svenson (1994) extended Nelson-Siegel model by introducing additional parameters permitting yield curve to have an additional hump.

$$f_t(\tau) = \beta_{0t} + \beta_{1t}e^{-\lambda_{1t}\tau} + \beta_{2t}\lambda e^{-\lambda_{1t}\tau} + \beta_{3t}\tau\lambda e^{-\lambda_{1t}\tau}$$

The corresponding yield to maturity is of the form:

$$y_t(\tau) = \beta_{0t} + \beta_{1t} \left[\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} \right] + \beta_{2t} \left[\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau} \right] + \beta_{3t} \left[\frac{1 - e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau} \right]$$

The two additional parameters β_{3t} and λ_{2t} explain the extended robustness of the Svenson approach. The linear parameter defines the convexity or concavity of the second hump of the spot interest rate curve and the non-linear parameter λ_{2t} like in the λ_t of Nelson-Siegel model

II The Model

The Nelson and Siegel model (1987) was modified by Diebold and Li (2006) and reformulated so as to fit the forward rate function at a given date. The Nelson-Siegel's assumes that forward rates can be

written in the form

$$\begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} 1 \\ e^{-\lambda_t \tau} \\ \lambda_t \tau e^{-\lambda_t \tau} \end{pmatrix} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix}$$

and then functionally derived as:

$$f_t(\tau) = \beta_{0t} + \beta_{1t} e^{-\lambda_t \tau} + \beta_{2t} \tau \lambda_t e^{-\lambda_t \tau}$$

Where $\beta_{0t}, \beta_{1t}, \beta_{2t}$ are parameters. Invoking the mean value theorem, it is clear that, the yield curve as a function of maturity is formulated as below.

$$\text{Recall that } y_t(\tau) = \frac{1}{\tau} \int_0^\tau f_t(u) du$$

$$y_t(\tau) = \frac{1}{\tau} \int_0^\tau \beta_{0t} + \beta_{1t} e^{-\lambda_t u} + \beta_{2t} \tau \lambda_t e^{-\lambda_t u} du$$

$$y_t(\tau) = \beta_{0t} + \beta_{1t} \left[\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right] + \beta_{2t} \left[\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right]$$

where again $\beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda_t$ are parameters.

Following Berec (2010);

$$\lim_{\tau \rightarrow 0} y_\tau(t) = \beta_{0t} + \beta_{1t} \quad \text{and}$$

$$\lim_{\tau \rightarrow \infty} y_\tau(t) = \beta_{0t}$$

This implies that the short rate of $y_t(\tau)$ is $\beta_{0t} + \beta_{1t}$ while β_{0t} is the long term coefficient contributing to the yield.

The forward rate function $f_\tau(t)$ as observed approaches same limiting value as spot rate.

(Nelson and Siegel, 1987; Ying, and Linlin, 2014) observe that the shape flexibility could be explained in varying forms to appraise the parameters of the model. It is generally agreed among financial market experts that the parameters contribute the short-term, medium-term and long-term part

of the forward rate curve. β_{0t} contributes to the long-term component, the contribution of the short-term segment is β_{1t} while β_{2t} is the contribution of the medium-term segment. Nelson and Siegel (1987), therefore argues that with the correct selection of components weightings, a series of forward rate graphs having monotonically humped shape could be generated. The model can be written in $N \times 3$ matrix array in order to compute the parameters, given the time to maturity for distinct Nigerian Eurobond maturities $\tau_1, \tau_2, \tau_3, \tau_4, \dots, \tau_{n-1}, \tau_n$ and the corresponding yield to maturity as $y_t(\tau_1), y_t(\tau_2), y_t(\tau_3), \dots, y_t(\tau_{n-1}), y_t(\tau_n)$. Using the model described in Berec (2010), we obtain the optimal parameters of the model that is $\beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda_t$ in terms of best fitting as follows.

$$\begin{pmatrix} 1 & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & -e^{-\lambda_t \tau_1} \\ 1 & \frac{1 - e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \frac{1 - e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & -e^{-\lambda_t \tau_2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & \frac{1 - e^{-\lambda_t \tau_n}}{\lambda_t \tau_n} & \frac{1 - e^{-\lambda_t \tau_n}}{\lambda_t \tau_n} & -e^{-\lambda_t \tau_n} \end{pmatrix} \begin{pmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{pmatrix} = \begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ y_t(\tau_3) \\ \cdot \\ y_t(\tau_{n-1}) \\ y_t(\tau_n) \end{pmatrix}$$

$$y_t(\tau_k) = Y \lambda_t \beta_\tau, K = 0,1,2$$

Where $y_t(\tau_k)$ is an N-dimensional vector, Y_{λ_t} is $N \times 3$ and β_t is a 3-dimensional vector thus

$$y_t(\tau_k) = \begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ y_t(\tau_3) \\ \vdots \\ y_t(\tau_{n-1}) \\ y_t(\tau_n) \end{pmatrix},$$

$$Y\lambda_\tau = \begin{pmatrix} 1 & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & \frac{1 - e^{-\lambda_t \tau_1}}{\lambda_t \tau_1} & -e^{-\lambda_t \tau_1} \\ 1 & \frac{1 - e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & \frac{1 - e^{-\lambda_t \tau_2}}{\lambda_t \tau_2} & -e^{-\lambda_t \tau_2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & \frac{1 - e^{-\lambda_t \tau_n}}{\lambda_t \tau_n} & \frac{1 - e^{-\lambda_t \tau_n}}{\lambda_t \tau_n} & -e^{-\lambda_t \tau_n} \end{pmatrix},$$

$$\beta_t = \begin{pmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{pmatrix}$$

In order to find solution to the system parameters above using the vector equations $Y\lambda_\tau$ and β_t , Nelson-Siegel suggested that for any $\lambda_\tau \geq 0$, the parameters may be estimated by applying ordinary least square method. Following Berc (2010), large values of λ_τ could be associated with fast decay in the regressors and consequently can fit extreme curvature at low maturities but may not be able to fit extreme curvature at high maturity interval. The author further argues that Low values of λ_τ correspondingly generates slow decay

function in the regressors which may fit curvature at higher maturities, however they may not work for very high curvature at low maturities. In order to avoid the processes of repeating different values of λ_τ for the best fit of the parameters, (Cox, Ingersoll & Ross, 1985; Diebold & Li, 2006; Berc, 2010) suggested the following optimization condition

$$\lambda^x = \max \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right)$$

3. Results and Discussion

3.1 Data Analysis

The database presented in this paper comprises of daily closing of the Nigerian Eurobond from January to May 2019 obtained from the Debt Management Office website to analyse and fit the Nigerian Eurobond yield curve using the Nelson-Siegel model (1987).

3.1.1: Monthly Analysis of the Yield:

Table 1. Descriptive Statistics for January 2019

Tenor(τ) in years	Mean yield	max. yield	min. yield	standard deviation	Coefficient of variation
2	5.654111	6.130	4.970	0.374492805	0.066233720
3	6.051222	6.598	5.424	0.379891680	0.062779334
4	6.660889	6.381	5.932	0.488453540	0.073331584
6	7.515556	8.266	6.862	0.489541651	0.065137117
8	7.763444	8.387	7.149	0.425147360	0.054762726
10	8.214111	8.783	7.620	0.391773992	0.047695240
12	8.557111	9.150	7.966	0.399250687	0.046657182
13	8.503778	9.072	7.888	0.395175207	0.046470546
20	8.596778	9.095	8.075	0.340834923	0.039646822
23	8.675111	9.194	8.158	0.352703999	0.040657001
30	9.106778	9.550	8.641	0.299846952	0.032925690

The Table 1 above shows a statistical summary of yield and the corresponding time to maturity for the month of January 2019, therefore it is understood that the higher the time to maturity, the higher the yield. Also, the longest-term yield (30 years) has a lower volatility than the short-term yield with the higher volatility in the 6 years maturity yield. Critical to mention is the lower yield of 13 years yield to maturity as compared to 12 years yield to maturity despite the fact that higher maturity should have a higher yield in comparison.

Table 2. Descriptive statistics for February 2019

Tenor(τ) in years	Mean yield	Max. yield	Min. yield	Standard deviation	Coefficient of variation
2	5.000000	5.215	4.654	0.191543	0.038308572
3	5.454143	5.597	5.145	0.165635	0.030368579
4	5.953857	6.103	5.572	0.193945	0.032574745
6	6.783286	6.970	6.447	0.176699	0.026049190
8	7.081857	7.277	6.749	0.185131	0.026141585
10	7.490429	7.687	7.151	0.188875	0.025215461
12	7.864000	8.036	7.543	0.173615	0.022077126
13	7.820429	7.988	7.559	0.152546	0.019506094
20	8.067714	8.232	7.807	0.151682	0.018801170
23	8.117714	8.292	7.875	0.140253	0.017277401
30	8.571857	8.762	8.254	0.174415	0.020347360

In the month February 2019, similar trend is observed as established in the descriptive analysis of January above. Furthermore, the longest-term yield (30 years) has a lower volatility than the short-term yield with the higher volatility in the 6 years maturity yield. The volatility measure in February is relatively low as compared to that of

January. However, there was a higher yield in the month January compared to February and also the subsequent months. There is a comparative decrease in yield of 13 years yield to 12 years yield to maturity despite the fact that higher maturity should have a higher yield, is also observed.

Table 3. Descriptive Statistics March 2019

Tenor(τ) in years	Mean yield	Max. yield	Min. yield	Standard deviation	Coefficient of variation
2	4.709778	4.831	4.639	0.056839	0.012068340
3	5.133222	5.268	4.994	0.099356	0.019355562
4	5.377778	5.577	5.03	0.18308	0.034043882
6	6.392778	6.584	6.238	0.140978	0.022052642
8	6.713222	6.88	6.564	0.115747	0.017241695
10	7.137111	7.273	6.973	0.105942	0.014843757
12	7.440111	7.628	7.249	0.134616	0.018093225
13	7.431000	7.646	7.233	0.145509	0.019581293
20	7.806000	7.945	7.636	0.108258	0.013868536
23	7.933778	8.058	7.772	0.097653	0.012308568
30	8.284778	8.401	8.119	0.099121	0.011964214

In the month of March, There is a decline in the yield of March compared to the month of January and February. The month of March also witness the relative decline of yield in 13 years' time to maturity compared to 12 years' time to maturity which has a higher

yield. The comparative volatility between the short-term maturity and the long-term maturity also showed a higher volatility on short-term yield in relation to the long-term yield. The maturity with the highest volatility is 13 years' time to maturity.

Table 4. Descriptive statistics for April 2019

Tenor(τ) in years	Mean yield	Max. yield	Min. yield	Standard deviation	Coefficient of variation
2	4.71	4.811	4.635	0.01672	0.00355
3	5.11	5.193	5.042	0.01632	0.00320
4	5.45	5.536	5.383	0.01467	0.00269
6	6.28	6.372	6.231	0.01261	0.00201
8	6.70	6.786	6.658	0.01389	0.00207
10	7.11	7.234	7.019	0.02212	0.00311
12	7.41	7.500	7.338	0.01605	0.00216
13	7.39	7.490	7.314	0.01848	0.0025
20	7.77	7.861	7.700	0.0162	0.00208
23	7.91	8.028	7.806	0.02814	0.00356
30	8.34	8.464	8.236	0.02596	0.00311

Just as in the previous months, there is an increase in yield in April compared with that of March. The decline in yield on 13 years to maturity in relation to 12 years' time to

maturity was also observed. The volatility comparison also showed lower volatility in long-term to short-term.

Table 5. Descriptive Statistics for May 2019

Tenor(τ) in years	Mean yield	Min. yield	Max. yield	Standard deviation	Coefficient of variation
2	4.77	4.639	4.928	0.09497	0.01993
3	5.13	4.991	5.207	0.05853	0.01142
4	5.54	5.466	5.654	0.05446	0.00982
6	6.40	6.258	6.513	0.0844	0.01319
8	6.90	6.717	7.030	0.10708	0.01553
10	7.34	7.154	7.477	0.10914	0.01487
12	7.60	7.401	7.740	0.11896	0.01566
13	7.59	7.12	7.768	0.18071	0.02381
19	7.94	7.804	8.077	0.08571	0.01079
23	8.16	7.993	8.319	0.1013	0.01242
30	8.54	8.422	8.689	0.08156	0.00955

This month witnesses a relative increase in yield compared to that of March and April. The decline in yield of 13 years to maturity in relation to 12 years' time to maturity was observed just as in the previous months. The

volatility comparison also showed lower volatility in long-term to short-term. There is an increase in volatility in the month of May compared to April

Aggregate descriptive statistics

Table 6. Summary Descriptive Statistics

Time to maturity(τ) in months	N	Mini yield	Max yield	Mean yield	Std. Deviation
24 months	5	4.71	5.65	4.9682	.40157
36 months	5	5.11	6.05	5.3742	.40503
48 months	5	5.38	6.66	5.7965	.53243
72 months	5	6.28	7.52	6.6743	.50724
96 months	5	6.70	7.76	7.0304	.43858
120 months	5	7.11	8.21	7.4574	.45090
144 months	5	7.41	8.56	7.7740	.47310
156 months	5	7.39	8.50	7.7473	.45539
240 months	5	7.77	8.60	8.0364	.33450
276 months	5	7.91	8.68	8.1577	.30936
360 months	5	8.28	9.11	8.5694	.32495
Valid N (listwise)	5				

IV ESTIMATION OF THE NELSON-SIEGEL PARAMETERS

Table 7. Time varying parameters of Nelson-Siegel model

Month	Level (β_{0t})	Slope (β_{1t})	Curvature (β_{2t})
January	9.405	-5.950	-0.270
February	8.915	-5.576	-1.594
March	8.759	-5.084	-3.277
April	8.770	-4.914	-3.660
May	9.014	-5.289	-3.590

Table 7 above showed the parameters estimated by linear regression of the Nelson-Siegel model for the 5 months from January to May 2019. The parameter β_{0t} which is the constant and defined economically as the long-term rate for the month of January, February, March, April and May are 9.405, 8.915, 8.759, 8.770 and 9.014 respectively are significant as it shows the long-term yield (30 years to maturity). The

second parameter β_{1t} for the month of January, February, March, April and May are -5.950, -5.576, -5.084, -4.914 and -5.289 respectively and also significant. Its negative sign indicates that as time increases, the parameter with the factor loading increases in value and leads to a reduction effect on the long rate factor (β_{0t}). The third parameter β_{2t} for the month of

January, February, March, April and May are:

-0.270, -1.594, -3.277, -3.660, -3.590 respectively are also significant having the

same effect on the model just as the parameter β_{1t} with a relatively lower reduction capacity.

Table 8. Aggregate coefficient

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta		
	β_{0t}	8.787	.134		65.658	.000
	β_{1t}	-7.634	.438	-1.081	-17.416	.000
	β_{2t}	1.776	1.062	.104	1.672	.133

The modified Nelson-Siegel, explained the first parameter β_{0t} as long-term yield and $\beta_{0t} + \beta_{1t}$ as a short-term yield. From the monthly analysis of the Nelson-Siegel model, we can define β_{0t} as long-term yield. Also, the short-term yield can be defined as $\beta_{0t} + \beta_{1t}$ assuming short-term here is 3 months maturity. The R^2 adjusted

determine how well the model fits into the observed data. The Nelson-Siegel model for this study fits in very well with the observed data which has the aggregate R^2 adjusted of 0.989 as shown in the table 9 below. This value signifies that 98.9% of the observed data can be explained by the model.

Table 9: Nelson-Siegel Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.995 ^a	0.991	0.989	0.14628	0.991	440.09	2	8	0

Given the value of the parameters $\beta_{0t}, \beta_{1t}, \beta_{2t}, \lambda$, the in-sample estimate for maturities (τ) not available in the observed data may be interpolated using the model. For each month of study, the following model is derived.

The Nelson-Siegel lamda was computed by solving the non-constraint optimization problem

$$\lambda^x = \max \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right), \text{ the resulting value is } \lambda_t = 0.03778438.$$

$$\begin{aligned} \text{January} = y_{\tau}(t) &= 9.405 - 5.950 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 0.270 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \\ \text{February} = y_{\tau}(t) &= 8.195 - 5.576 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 1.594 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \\ \text{March} = y_{\tau}(t) &= 8.759 - 5.084 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 3.277 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \\ \text{April} = y_{\tau}(t) &= 8.770 - 4.914 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 3.660 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \\ \text{May} = y_{\tau}(t) &= 9.014 - 5.289 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 3.590 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \\ \text{aggregate} = y_{\tau}(t) &= 8.787 - 7.634 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} \right] - 1.776 \left[\frac{1 - e^{-0.03778438\tau}}{0.03778438} - e^{-0.03778438\tau} \right] \end{aligned}$$

V Discussion of Findings

The results of the Nelson-Siegel model at a specified point in time and as the time to maturity (τ) approaches zero describes the limiting value of the yield as short-term rate $\beta_{0t} + \beta_{1t}$ such as 3 months. A major constraint we have is that, the data available for this study does not provide for such time to maturity, despite this, the resulting numerical values reveal a low yield which could be attributed to short-term yield. The result further reveals that as time to maturity tends to infinity, the resulting long-term yield for the 5 months are 9.405, 8.915, 8.759, 8.770 and 9.014 respectively showing the observed long-term maturity. From the positive slope of the Nigerian Eurobond yield curve over the five months in this paper, we can infer that returns on investment on Nigerian Eurobond is anticipated to increase as time to maturity rises. The yield curve positive slope demonstrates the market expectation that

subsequently, the higher yield on long-term maturities could sufficiently favour the market makers over the investment risk horizon. This is an incentive for investors to invest Nigerian economy in anticipation of reward return. Despite the positive slope of yield curve justifying market makers' readiness to invest, however, risk uncertainties may occur where they decide on improved investment opportunity in anticipation of adverse economic conditions in the future.

VI Conclusions

The objective is to appraise time varying parameters and curvature of Nigerian Eurobond on

Nelson-Siegel model (1987) so as to fit in the observed data and then estimate the parameters. Our results confirms that the betas of the term structure of interest rate volatilities are functionally related when appraising the term structure of Euro-bond yield and which the Nelson-Siegel model provides a comfortable solution. The paper

has clearly shown the level at which the Nelson-Siegel model can fit into the Nigerian Eurobond yield for the five months of study (January, February March, April and May 2019) with a high adjusted value R^2 of 98% as a measure of fit. The model parameters were estimated to investigate if the interpretation by modified Nelson-Siegel are applicable on the observed data yield at differing time to maturities. This yield curve can serve as a good curve to choose measurable discount rate for insured schemes such as retirement benefit scheme from actuarial perspective.

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